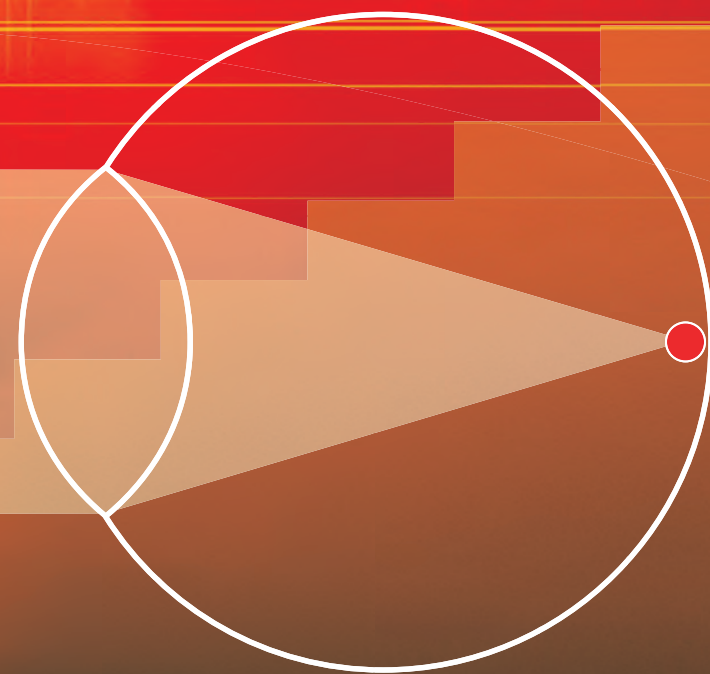# Curriculum Focal Points

for Prekindergarten *through* Grade 8 Mathematics

A Quest for Coherence

NCTM
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Curriculum Focal Points and Connections for Prekindergarten

The set of three curriculum focal points and related connections for mathematics in prekindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Prekindergarten Curriculum Focal Points	Connections to the Focal Points
Number and Operations: **Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison** Children develop an understanding of the meanings of whole numbers and recognize the number of objects in small groups without counting and by counting—the first and most basic mathematical algorithm. They understand that number words refer to quantity. They use one-to-one correspondence to solve problems by matching sets and comparing number amounts and in counting objects to 10 and beyond. They understand that the last word that they state in counting tells "how many," they count to determine number amounts and compare quantities (using language such as "more than" and "less than"), and they order sets by the number of objects in them.	*Data Analysis:* Children learn the foundations of data analysis by using objects' attributes that they have identified in relation to geometry and measurement (e.g., size, quantity, orientation, number of sides or vertices, color) for various purposes, such as describing, sorting, or comparing. For example, children sort geometric figures by shape, compare objects by weight ("heavier," "lighter"), or describe sets of objects by the number of objects in each set. *Number and Operations:* Children use meanings of numbers to create strategies for solving problems and responding to practical situations, such as getting just enough napkins for a group, or mathematical situations, such as determining that any shape is a triangle if it has exactly three straight sides and is closed.
Geometry: **Identifying shapes and describing spatial relationships** Children develop spatial reasoning by working from two perspectives on space as they examine the shapes of objects and inspect their relative positions. They find shapes in their environments and describe them in their own words. They build pictures and designs by combining two- and three-dimensional shapes, and they solve such problems as deciding which piece will fit into a space in a puzzle. They discuss the relative positions of objects with vocabulary such as "above," "below," and "next to."	*Algebra:* Children recognize and duplicate simple sequential patterns (e.g., square, circle, square, circle, square, circle,…).
Measurement: **Identifying measurable attributes and comparing objects by using these attributes** Children identify objects as "the same" or "different," and then "more" or "less," on the basis of attributes that they can measure. They identify measurable attributes such as length and weight and solve problems by making direct comparisons of objects on the basis of those attributes.	

Curriculum Focal Points and Connections for Kindergarten

The set of three curriculum focal points and related connections for mathematics in kindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Kindergarten Curriculum Focal Points	Connections to the Focal Points
Number and Operations: Representing, comparing, and ordering whole numbers and joining and separating sets Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.	**Data Analysis:** Children sort objects and use one or more attributes to solve problems. For example, they might sort solids that roll easily from those that do not. Or they might collect data and use counting to answer such questions as, "What is our favorite snack?" They re-sort objects by using new attributes (e.g., after sorting solids according to which ones roll, they might re-sort the solids according to which ones stack easily).
Geometry: Describing shapes and space Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.	**Geometry:** Children integrate their understandings of geometry, measurement, and number. For example, they understand, discuss, and create simple navigational directions (e.g., "Walk forward 10 steps, turn right, and walk forward 5 steps"). **Algebra:** Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.
Measurement: Ordering objects by measurable attributes Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.	

Curriculum Focal Points and Connections for Grade 1

The set of three curriculum focal points and related connections for mathematics in grade 1 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 1 Curriculum Focal Points	Connections to the Focal Points
Number and Operations and ***Algebra:*** **Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts** Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number lines, to model "part-whole," "adding to," "taking away from," and "comparing" situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.	***Number and Operations*** and ***Algebra:*** Children use mathematical reasoning, including ideas such as commutativity and associativity and beginning ideas of tens and ones, to solve two-digit addition and subtraction problems with strategies that they understand and can explain. They solve both routine and nonroutine problems. ***Measurement*** and ***Data Analysis:*** Children strengthen their sense of number by solving problems involving measurements and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships.
Number and Operations: **Developing an understanding of whole number relationships, including grouping in tens and ones** Children compare and order whole numbers (at least to 100) to develop an understanding of and solve problems involving the relative sizes of these numbers. They think of whole numbers between 10 and 100 in terms of groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and their relative magnitudes and represent numbers on a number line.	***Algebra:*** Through identifying, describing, and applying number patterns and properties in developing strategies for basic facts, children learn about other properties of numbers and operations, such as odd and even (e.g., "Even numbers of objects can be paired, with none left over"), and 0 as the identity element for addition.
Geometry: **Composing and decomposing geometric shapes** Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry.	

Curriculum Focal Points and Connections for Grade 2

The set of three curriculum focal points and related connections for mathematics in grade 2 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 2 Curriculum Focal Points	Connections to the Focal Points
***Number and Operations:* Developing an understanding of the base-ten numeration system and place-value concepts** Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).	***Number and Operations:*** Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition.
***Number and Operations* and *Algebra:* Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction** Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.	***Geometry* and *Measurement:*** Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions. ***Algebra:*** Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.
***Measurement:* Developing an understanding of linear measurement and facility in measuring lengths** Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).	

Curriculum Focal Points and Connections for Grade 3

The set of three curriculum focal points and related connections for mathematics in grade 3 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 3 Curriculum Focal Points	Connections to the Focal Points
Number and Operations and ***Algebra:*** **Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts** Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.	***Algebra:*** Understanding properties of multiplication and the relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level. Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, "The number of legs is 4 times the number of chairs."
Number and Operations: **Developing an understanding of fractions and fraction equivalence** Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.	***Measurement:*** Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two-dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.
Geometry: **Describing and analyzing properties of two-dimensional shapes** Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.	***Data Analysis:*** Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems. ***Number and Operations:*** Building on their work in grade 2, students extend their understanding of place value to numbers up to 10,000 in various contexts. Students also apply this understanding to the task of representing numbers in different equivalent forms (e.g., expanded notation). They develop their understanding of numbers by building their facility with mental computation (addition and subtraction in special cases, such as 2,500 + 6,000 and 9,000 – 5,000), by using computational estimation, and by performing paper-and-pencil computations.

Curriculum Focal Points and Connections for Grade 4

The set of three curriculum focal points and related connections for mathematics in grade 4 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 4 Curriculum Focal Points	Connections to the Focal Points
Number and Operations **and** ***Algebra:*** **Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication** Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.	***Algebra:*** Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects. ***Geometry:*** Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons. They build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. By using transformations to design and analyze simple tilings and tessellations, students deepen their understanding of two-dimensional space.
Number and Operations: **Developing an understanding of decimals, including the connections between fractions and decimals** Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.	***Measurement:*** As part of understanding two-dimensional shapes, students measure and classify angles. ***Data Analysis:*** Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots.
Measurement: **Developing an understanding of area and determining the areas of two-dimensional shapes** Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.	***Number and Operations:*** Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts. They use estimation in determining the relative sizes of amounts or distances. Students develop understandings of strategies for multidigit division by using models that represent division as the inverse of multiplication, as partitioning, or as successive subtraction. By working with decimals, students extend their ability to recognize equivalent fractions. Students' earlier work in grade 3 with models of fractions and multiplication and division facts supports their understanding of techniques for generating equivalent fractions and simplifying fractions.

Curriculum Focal Points and Connections for Grade 5

The set of three curriculum focal points and related connections for mathematics in grade 5 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 5 Curriculum Focal Points	Connections to the Focal Points
***Number and Operations* and *Algebra:* Developing an understanding of and fluency with division of whole numbers** Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems. They consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.	***Algebra:*** Students use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities. They create graphs of simple equations. They explore prime and composite numbers and discover concepts related to the addition and subtraction of fractions as they use factors and multiples, including applications of common factors and common multiples. They develop an understanding of the order of operations and use it for all operations. ***Measurement:*** Students' experiences connect their work with solids and volume to their earlier work with capacity and weight or mass. They solve problems that require attention to both approximation and precision of measurement.
***Number and Operations:* Developing an understanding of and fluency with addition and subtraction of fractions and decimals** Students apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They apply their understandings of decimal models, place value, and properties to add and subtract decimals. They develop fluency with standard procedures for adding and subtracting fractions and decimals. They make reasonable estimates of fraction and decimal sums and differences. Students add and subtract fractions and decimals to solve problems, including problems involving measurement.	***Data Analysis:*** Students apply their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on coordinate grids. ***Number and Operations:*** Building on their work in grade 4, students extend their understanding of place value to numbers through millions and millionths in various contexts. They apply what they know about multiplication of whole numbers to larger numbers. Students also explore contexts that they can describe with negative numbers (e.g., situations of owing money or measuring elevations above and below sea level).
***Geometry* and *Measurement* and *Algebra:* Describing three-dimensional shapes and analyzing their properties, including volume and surface area** Students relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces. Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. They understand that a cube that is 1 unit on an edge is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume. They decompose three-dimensional shapes and find surface areas and volumes of prisms. As they work with surface area, they find and justify relationships among the formulas for the areas of different polygons. They measure necessary attributes of shapes to use area formulas to solve problems.	

Curriculum Focal Points and Connections for Grade 6

The set of three curriculum focal points and related connections for mathematics in grade 6 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 6 Curriculum Focal Points	Connections to the Focal Points
Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement. **Number and Operations: Connecting ratio and rate to multiplication and division** Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates. **Algebra: Writing, interpreting, and using mathematical expressions and equations** Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as $3x = y$) shown in a table.	**Number and Operations:** Students' work in dividing fractions shows them that they can express the result of dividing two whole numbers as a fraction (viewed as parts of a whole). Students then extend their work in grade 5 with division of whole numbers to give mixed number and decimal solutions to division problems with whole numbers. They recognize that ratio tables not only derive from rows in the multiplication table but also connect with equivalent fractions. Students distinguish multiplicative comparisons from additive comparisons. **Algebra:** Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are $x + 3$ by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas. **Measurement** and **Geometry:** Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students' work in grade 5 on area and volume and provide a context for applying new work with equations.

Curriculum Focal Points and Connections for Grade 7

The set of three curriculum focal points and related connections for mathematics in grade 7 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 7 Curriculum Focal Points	Connections to the Focal Points
Number and Operations and _Algebra_ and _Geometry:_ Developing an understanding of and applying proportionality, including similarity Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$). **_Measurement_ and _Geometry_ and _Algebra:_ Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes** By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (_Volume = Area of base × Height_). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three-dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders. **_Number and Operations_ and _Algebra:_ Developing an understanding of operations on all rational numbers and solving linear equations** Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.	**_Measurement_ and _Geometry:_** Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings. **_Number and Operations:_** In grade 4, students used equivalent fractions to determine the decimal representations of fractions that they could represent with terminating decimals. Students now use division to express any fraction as a decimal, including fractions that they must represent with infinite decimals. They find this method useful when working with proportions, especially those involving percents. Students connect their work with dividing fractions to solving equations of the form $ax = b$, where a and b are fractions. Students continue to develop their understanding of multiplication and division and the structure of numbers by determining if a counting number greater than 1 is a prime, and if it is not, by factoring it into a product of primes. **_Data Analysis:_** Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs. **_Probability:_** Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.

Curriculum Focal Points and Connections for Grade 8

The set of three curriculum focal points and related connections for mathematics in grade 8 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 8 Curriculum Focal Points	Connections to the Focal Points
Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize a proportion ($y/x = k$, or $y = kx$) as a special case of a linear equation of the form $y = mx + b$, understanding that the constant of proportionality (k) is the slope and the resulting graph is a line through the origin. Students understand that the slope (m) of a line is a constant rate of change, so if the input, or x-coordinate, changes by a specific amount, a, the output, or y-coordinate, changes by the amount ma. Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and y-intercept appear in different representations. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.	**Algebra:** Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed. **Geometry:** Given a line in a coordinate plane, students understand that all "slope triangles"—triangles created by a vertical "rise" line segment (showing the change in y), a horizontal "run" line segment (showing the change in x), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line.
Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.	**Data Analysis:** Building on their work in previous grades to organize and display data to pose and answer questions, students now see numerical data as an aggregate, which they can often summarize with one or several numbers. In addition to the median, students determine the 25th and 75th percentiles (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information. Students make scatterplots to display bivariate data, and they informally estimate lines of best fit to make and test conjectures. **Number and Operations:** Students use exponents and scientific notation to describe very large and very small numbers. They use square roots when they apply the Pythagorean theorem.
Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.	