MUIR: My name's Allen Muir. I'm from PaTTAN Harrisburg, and I have the pleasure of introducing this afternoon's speaker, Dr. Douglas Clements. He's widely regarded as the major scholar in the field of early childhood mathematics education. At the national level, his contributions have led to the development of new mathematics curricula, teaching approaches, teaching training initiatives, and models of scaling up interventions as well as having a tremendous impact on educational planning and policy, particularly in the area of mathematical literacy and access.

He has served on the President's National Mathematics Advisory Panel, the National Research Council's Committee on Early Mathematics, the National Council of Teachers of Mathematics National Curriculum, and Principles and Standards Committees, and is a co-author of each of their reports. He is presently serving on the Common Core Committees of National Governors' Association and the Council of Chief State School Officers helping to write national academic standards.

A prolific and widely-cited scholar, he has earned external grant support totaling nearly $19 million, including major grants from the National Science Foundation, the National Institute of Health, and the National Institute of Education Sciences of the U.S. Department of Education. Let's give a warm welcome to Dr. Clements.

CLEMENTS: Thank you. Thank you very much. Appreciate it. Not always widely-regarded for sure. I'll tell you a story about my first year in teaching kindergarten. Okay. Very first year, very first teaching experience I had other than student teaching and stuff like that. I was told that I had to do an afternoon tea. Now what they meant by that is the parents come after school, and you tell them about your program. It's about five weeks into the program, and they warn you, we warn the parents not to ask about your kids because that's for parent-teacher conferences ten weeks out, right, so don't worry about that.

But, of course, lots of people came up and talked to me afterwards about the kids, but I remember Jeanette's parents because they came up and said, oh, Mr. Clements, thank goodness Jeanette's doing better now. I'm thinking, there was nothing wrong with Jeanette, like she wasn't the one that left that first day, got 100 yards down the road before I caught him. That was a boy, and he was fast. I remember him. So I'm thinking, I'm trying to stall, I say, well, tell me more about that, kind of like you'd say to the kindergarten kids who bring their first painting to you, and it's one solid color, you know, tell me more about that.

And they said, well, she came home, and her dad said, tell us about your teacher and she wouldn't talk. Then Thursday, the same thing, Friday, the same thing, on Monday, her father sat down with her at dinner and got frustrated and said, Jeanette, you have been in school for four days, tell us about your teacher. She started crying and said, I can't, she keeps sending her husband in. So, so, from then on, I sent notes home, letters home with my picture and saying, really, I'm your teacher. You know, I'm sorry. Try for a woman the next grade, you know, or something.

So I've been in love with early childhood and early childhood mathematics education since then because I was just delighted with how much the kids could learn and how well they could think and the like. Lately, let me give you an illustration from one of, Julie Sarama, my wife and colleague's, activities in the Building Blocks, now this isn't the Building Blocks activity. These are kids who have engaged in Building Blocks during the year. They're four-year-olds. One's just turned five.

They're trying to finish a puzzle. It's just a puzzle they pulled out, not one of ours. It's just a wooden puzzle they pulled off the shelf, but it was interesting what they did with it. So the boy, Cory, kept putting four triangles together to make a square and then putting that down and then four more triangles to make another square and putting that down, illustrated so you can see it a little better in the upper right. And that was good.

This boy in the pink tried to duplicate what he was doing, but he just used two triangles together to make a smaller square, right, and it didn't fit. And they got all upset about it, and they talked it through, and Cory explained you had to have four triangles, you put them together like this, then it fits. And they finished the puzzle. The teacher, who was snapping the pictures, asked Cory, how many triangles did you use? And Cory went one, two, three, four, five, and counted very good for a four-year-old up to 24. And then she asked him 24 what? He said triangles.

How many squares did you have, she said, because she had seen what he was doing, and this is what's really interesting to Julie and me because he curled his little pinky up, put four fingers down on each of the four triangles that constituted a square, and counted one, two, three, four, five, six. So what's interesting to us, and I hope is interesting to you as people interested in mathematics, is that in two domains, the most two important domains for early mathematics, geometry and number, these boys had created units of units.

In other words, they put four triangles together to make a square and could see it simultaneously as four triangles and one square, and they could also count singletons and count some of those singletons together. When you think about, you people who are in second grade, and you're working with multi-digit addition and subtraction, and you need to have kids think of ten as ten 1's and one 10 simultaneously, you know how important that thinking is, right? So how do we get there?

Well, it's important to get there, and it's important to get there, anybody that unfortunately had to sit through my talk this morning and now is back with me knows that I made a big deal out of inequities between low resource and higher resource communities. Here in this talk I want to emphasize how early those inequities start. Okay. Here's two kids in a research study. Peter could count at the highest level. He could count beyond 120. He knew the number word before and the number word after every counting number up to the 100.

He could do addition and subtraction with very simple counting strategies. And there's another little boy, Tom, couldn't count. He was asked what number comes after six. He said horse. When . . . said, well, how about what number comes after one, he said bike. Both those boys are beginning kindergarten in the same classroom. This is in Australia, actually. This is where this research came from.

There's a three-year difference, a three-year difference between those kids before they're four years old, there's already a three-year difference in the preparation of some kids to the others, which means we really have to understand how these things develop, bring up those kids, provide really good experiences to those kids because initial knowledge of mathematics is remarkably persistent. The National Math Advisory

Panel found that you can use entering kindergarten scores to predict high school grades with some accuracy.

So kids start, kids who are behind start behind and they get increasingly behind as they go up through the grades. That gap progressively widens. So we need to do something about that. We need to start them right. We need to work hard with those kids, and which kids? The very, very poor. Well, a lot of those studies, that's what they do. They're contrasting these kids who are poor. But an interesting study shows that it's not just the very, very poor.

This is the bar of the highest quintile of kids in terms of income, parental income and other SES kind of markers. We can see that the gap is the most dramatic for the lowest income kids. But notice that that gap exists and is fairly substantial for every group except the highest quintile. So there's a lot of kids who start out a little bit behind those kids at the top who tend to be the ones that go on and be successful that we have to deal with.

So what do we do? Julie and I got a National Science Foundation project to work with curricula and that and look at this, the growth of mathematical ideas and how we can develop them in the early years. And we were funded by the National Science Foundation to do this Building Blocks project. A lot of the learning trajectories that I'm focused on today came from our work on that project, so let's just get it started by taking a little view of a classroom. We are lucky, actually, to get good video.

You'll see some of our video that we take, you know. And it's the kind of video where it's like this, you know, and you see the ceiling part of the time because it's us or our grad assistants taking it. In this case, people from Boston had called the National Math Advisory Panel when the report came out and said, your panel's report's depressing. You know, we don't do well internationally, we got lots of weaknesses. You know, there's too few people like yourselves working hard to make it better. Is anything going well?

And they said, well, you should try the Boston City Schools because they were the most improved lately and, thank goodness, they actually, the principal directed them to one of ours because we're doing a big scale-up project, research project. This is a preschool that they wandered into and took the video from in the Building Blocks project. Let's take a look. Little intro here. This isn't very long.

[Videotape played]

CLEMENTS: So a little bit funny and a lot sad, right. You know, it's common that, you may, guys may have talked about the same kind of thing. We would never brag in this country, you know, I can't read. I have to go to the computer and have the computer read for me. But math, that can be a joke. You'd never find that true in the East Asian countries that in those international comparisons are doing so well compared to us. Math is a sign of intelligence.

Nobody there says, I had to Google it, I don't know my math, you know, I'm not a math person, right. So it's a cultural thing that we have to address with some of this too. Two things that you might not immediately notice on the video because they go by fairly quickly. Right towards the end, she makes an interesting statement that really captures why learning trajectories, the focus of this talk, are so important. I can look at the kids and look at my curriculum and know this is where the kid is. Here are some activities that I can do to really push them. Did you hear that?

You know, she's just talking informally, but that's at the heart of formative assessment, heart of whatever you want to call it, individualist instruction or differentiating instruction, is knowing where the kid is, knowing where you go next, not just mathematically, but thinking-wise. What's the next level of thinking? And then you got to know what activities are going to get them to the next level of thinking? That's what we're going to emphasize today.

Learning trajectories are about the mathematics of children because it's not just the mathematical next step. It's the next thinking step for kids. How do they represent the ideas, and how do they think about them? And activities matched to that developmental level. The benefit of learning trajectories is because they're based on this kind of developmental research and math ed research.

We know they're in the capabilities of kids because it came from the kids. We know it provides a national, natural building block to the next level for the same reason. There's longitudinal and cross-sectional studies that show us that. And, finally, we know it provides the most important mathematical building blocks, ironically, because most of that research comes from fairly well-to-do kids, successful kids because they're easy. They're in university settings and the like where a lot of this research is done. These are the kids who are successful. That's the capabilities they have.

So that's where we developed our learning trajectories. Learning trajectories have three components. There's a goal that you want the kid to reach, addition and subtraction of multi-digit numbers for second or third grade, multiplication, and knowing your geometric shapes, spatial thinking. There's a developmental progression that constitutes levels of thinking through which kids pass on their way to achieving that goal. And then there's instructional activities matched to each one of those levels of thinking that build the mental actions and objects that constitute the next level of thinking.

Okay. Those three parts. So let's talk about counting. Let's start with counting. The first goal, accurate, confident, object counting. Okay. That's the goal. The developmental progression for that, well, let's think. Talk to the people next to you, around you, at the table and say, pretend you got a kid, oh, you can make him as young as you want or maybe just a kid from a low resource community that hasn't had much experience. Think of a kid who knows nothing about counting.

Then think of that kid a couple years later, good education, really good counter. Okay. What's the difference? What developed between those two periods in that kid's life? What develops with counting development? Go ahead. I'll give you just two minutes. Talk to the people at your table and try to explicate as much as you can. What develops when counting develops? What develops when counting develops? Let's share. Who wants to start? Sir, go ahead.

AUDIENCE MEMBER: Language.

CLEMENTS: What kind of language?

AUDIENCE MEMBER: Well, being able to say that the counting numbers is in the sequence and in order.

CLEMENTS: Absolutely. You need that, right. You need to know they're counting words. They're different from other kind of words, and you need to be able to say them in order. Excellent. What else? Oh, you'd be amazed, you guys, because you know math, you know, maybe you've worked with enough colleagues not to be surprised, but I'm always surprised and disappointed at how many early childhood teachers think that's it, right. I checked. He can say one, two, three, four, five, six, seven, eight, nine, ten, so I knew he was fine in counting.

But, of course, you guys know there's much more. Who's got something else to add to that?

AUDIENCE MEMBER: A one-to-one correspondence and now your numbers have a quantity behind them.

CLEMENTS: Good. And, again, some teachers less knowledgeable than yourself think one-to-one correspondence, and the way to develop that, and they're not wrong, is to get like those K-Mart kind of, you know, sheets that have like the babies over here and the adults over here, and you draw a line, one baby to the adult. That's one-to-one correspondence too, but it's different in counting, right. How does it work in counting?

AUDIENCE MEMBER: . . .

CLEMENTS: What does one-to-one correspondence like look like in counting? What are you keeping a one-to-one correspondence between?

AUDIENCE MEMBER: The number and like the object.

CLEMENTS: Yeah, yeah. It sounded like I was asking more than I was. A simple question. But listen, it is relevant. And I'll tell you why. Okay. Karen Fusion, in her research in 1978, way back to then, points out that there's two one-to-one correspondences in counting, and we can sometimes forget that. And teachers who realize that can be more insightful about where it's going wrong, if it's going wrong for a kid.

There's a one-to-one correspondence between my enunciation of the counting words, which you were just talking about, and my pointing at. And if that goes wrong, I can go like one, two, three, four, five, six, seven. Now I pointed to each finger perfectly, but my words were all over the place. And then there's the correspondence between the pointing act and the object. So that, I can keep one, two, three, four, five, but if I go one, two, three, four and miss one or hit one twice or anything else.

So there's two one-to-one correspondences and since one of them is ethereal, the counting words, you can't see it, you can't grab it, it's an especially hard one-to-one correspondence. It's no longer developed over the course of years. Good. Good one, good. Essential. Who else? Anything else, or do we have it already?

AUDIENCE MEMBER: I think understanding that the last number you count is a …

CLEMENTS: Ahh. That's key. So what did we hear so far? Over here, we heard the number words, one-to-one correspondence and then the last number word tells how many, right. That's a big one, too. Okay. Let's go back to Karen's research. She points out, and this is really interesting to me, I hope it is to you, that there's a big shift there. A lot of people say, I'll just tell the kids, say the last number word over again. But it's way more than that, right.

Because when we're counting, fingers is a bad case because they're pretty well-ordered, right. But still in all, it's an ordinal activity in the act of counting. In other words, I got to keep my number words in order, I got to keep the objects in order, I got to make sure I'm going one-to-one for every one in order. But then when I stop, I have to make what she calls a counting to cardinal shift. In other words, I got to stop with the ordering, start thinking of all of them, and switch from ordination, sequencing, to cardinality, how many-ness.

So when I get to the end, it's a big deal. And you think of it, how different from other words. So I can't see your name tag. I need, I know Natalie from before. What's your name?

AUDIENCE MEMBER: I'm Jane.

CLEMENTS: Jane. Thanks. I forgot.

AUDIENCE MEMBER: Rosette.

CLEMENTS: And Rosette. So if somebody asks me over there, who do you got over there? And I said Jane and Natalie and Rosette. And you said, well, bring Rosette over. Rosette, would you go over and work with them? But if they say, how many you have on that side of the table, and I said, I got three, and they said, send those three over here, I don't send Rosette.

So it's very different from the naming we do with other words, this number stuff, and it's a big deal for kids. Very important thing. Is that about it for counting then? That's all we got? Did we nail it with those one, two, three things?

AUDIENCE MEMBER: What about ordinal position?

CLEMENTS: How do you mean?

AUDIENCE MEMBER: Like first, second, third.

CLEMENTS: Yeah, yeah. So it's not just, I mean, if you use the ordinal numbers as ordinal numbers, that's a different thing. It's connected to counting, but there's also this, when you're counting, knowing that three comes between different numbers, two and four, is an important thing too. I heard you guys, I think it was you guys, talking about more and less, and that's related to this ordinal thing too. And that's a part of counting. And yet there's much more.

So I bet you thought of more, but let's see some videos and then talk about that again. Okay. Here's two kids. Compare their counting for me and in the back of your mind, I also want you to think, this is how Julie and I, my wife and colleague, and I always say that, and people say, why do you keep saying wife and colleague? Because I used to say she's, my colleague and stuff, and then I'd say, these are our kids and stuff. And she said, what do you mean and stuff? And you talk about our kids and then you say, and stuff, what do you mean by that? So I'm very careful now, wife and colleague, wife and colleague.

Julie and I use these videos. We show them to teachers so that they can think about the different, the differences in counting, the skills that go into counting because they don't come up with it on their own all the time. So you'll recognize this one right away. Okay. Compare these two girls.

[Videotape played]

CLEMENTS: Compared to.

[Videotape played]

CLEMENTS: And so we asked the teachers, so what's the difference between those two? And, indeed, you guys have already analyzed it. What is the difference then?

AUDIENCE MEMBER: . . .

CLEMENTS: Right, right. I mean, right. Both girls, and that first girl was really good at what you said, and she was really good at what you said. She didn't seem to understand what you said, right. So she had one-to-one correspondence, she had the number words in order at least up to five in a line and stuff like that. She didn't seem to understand, and some people say, well, but when you ask her how many, and she just counted, maybe she gets concerned that she did it wrong or something.

This is only one of like four tasks we give the kids. She didn't understand that the last counting word said how many. Let me drive that home with one more thing. We were just talking about Number Worlds. Sharon Griffin and Robbie Case, the original authors of Number Worlds way back when they were just starting that research, their research found that of low income kids, a good minority, it's a minority, but a good, solid minority, didn't understand the cardinality principle in first grade.

So here they go, the teacher starts addition, and they don't understand that the last number word stands for how many in the whole set. What can they do? Well, Julie Sarama, my wife and colleague, directed a dissertation when she was at Wayne State University in Detroit, and all the research or the Ph.D. candidate did is ask first graders, what do you do when the teacher teaches mathematics? And one of the most poignant answers were a kid who said, I know what I do when she teaches math. I empty my mind, and I listen to the teacher.

You know, all of us have probably somewhere in our math career kind of wanted to empty our mind and say, don't explain it, just tell me what to do to get the answer. But that's a lousy way to start your mathematics education. If you got to just forget everything I know and tell me what to say, and I'll try rote memory, it does not work. And it just doesn't work for kids. They have to understand it, make sense of it, and the like. They can't make sense of it if they missed a step.

One of the beautiful things about learning trajectories, especially for kids with special needs, is that they kind of map out these levels of thinking so you can check did the kid get all these levels? Did they miss something on the way up? And then, what are instructional activities I can do to close that gap? Okay. So the learning trajectory for counting and you, if you care, you don't have to right now because it won't make any difference. But I have the learning trajectories, this second page in from your thing is just some Q&A on learning trajectories, not very important for today. Maybe you'll find it interesting.

The next page, and I think, right, is the learning trajectory for preschool through two in counting. Okay. And so you don't have to look at it now. It's not important. I just wanted you to have it. We're going to do some activities later where you have to figure out what level the kids is at, and then I want you in front of it, but go ahead. I forgot your name already. Rosette.

AUDIENCE MEMBER: If a child has dyscalculia, is there any specific things you could teach them that might normally be automatic?

CLEMENTS: Yeah, yeah. If a child has dyscalculia, what do you do? Are there things that, the counting sequence, arithmetic, facts which we talked a lot about this morning, but I'll try to talk a little more about today, have some particular steps and stuff like that that are real important for those kids that can help develop that. And then one more thing, which I think I have in this presentation, but now I can't remember off-hand, is subitizing, the quick and accurate recognition of small groups. That piece is missing in a lot of kids who have honest-to-God learning difficulties in mathematics.

As you probably know, about 80% of young kids are misidentified as having learning disabilities when actually what they have is no experience yet and no opportunities to learn. So you want to be careful, of course, setting that label on the kid. But when you know and you have these difficulties or if they just don't have these experiences, there are some of things. Ask me again if by half an hour in I don't, I haven't answered that completely. Okay. Because we'll come back to it. These are important things.

You want to make sure the kids can do all this. I mean, this isn't actually something we teach to. This is the pre-counter level where kids know that the number words are different. If you asked a kid, how many? They'll often say a number just out of a hat. But they'll use number words. Very few of them say horse or apple or something like that. They know they're special, but they can't keep them in order. Then they learn that to chant the number words, but there's often two things wrong.

These are the kids who are at the level of L, M, N, O, P or sweet land of liberty. The words are all bunched together, and they can't keep one-to-one correspondence because they don't know where one word ends and another one starts. Even when they know that, they sometimes will count one, two, three, four, five, six, se-ven. You know, so they need that differentiation of what these number words are. One of the advantages of East Asian languages, incidentally, is that their number words are way shorter than ours. Easy to pronounce, easy to remember. Never multi-syllabic.

So it's interesting. We got to just watch for that, that kids do that. We can't change the language, but we can know that that makes it particularly difficult for kids. Okay. So after that, then they get to this first important stage we call reciter. The names are not important. We just have teachers name them so that it would make sense to teachers, right. And so what's important is the ideas and how they develop. Here's where you can verbally count to five, then later to ten. You know where the word separate and you can keep them in a constant order.

You might fall apart in the teens. The research shows that most kids have a stable section. Then a kind of unstable section where it's kind of loose. And then random words come out at the top. And those sections just keep getting bigger and bigger in the good sense, right. Okay. Then we saw that first girl, both of them, but that first girl is at this level. She's at the level of the corresponder. She can keep one-to-one correspondence, she knows those number words in order. She does not know that the last number word means that.

Now be careful of the correspondence too. A caveat is it doesn't mean she can do correspondence in every situation. You know, you guys know that if you want to know how high a young kid can count, give them a bunch of objects arranged in a circle. They'll count and just keep going around, and you can see how high they can go, right. She couldn't do that yet, only in a line, but at least she has the idea. She just needs more experience generalizing that idea, right.

The second little girl was at least here, might have been higher. We didn't see that, but at least here. She can count one to five objects in a line meaningfully and really understands that the last number word means how many, the cardinality principle. Okay. Then there's more levels above that. We didn't name some of these, but maybe some of you guys were thinking about it. The producer can produce a set. That's harder. For adults, it doesn't feel harder. We can count out five. Big deal. But for kids, you think about it, there's no automatic way of knowing you have to, where you stop. You have to have a loop in your head.

Okay. Give me five. One, two, three, four, five, six. No, no, no, just five. Oh, yeah, yeah. One, two, three, four, five, six because their whole working memory is taken up just with producing the objects, keeping it in one-to-one correspondence with the production of a number word, and that takes their whole working memory. So then they just can't keep the loop going, checking each time if they've reached the end and stopping themself. They can eventually, but that takes a little extra work, and that' why for most kids, not every single kid follows this path. This is just the general path for most kids, but it's very important teachers develop this skill after this skill. Because if you don't and you skip it, again, there's a gap for kids. Okay.

Then we call it counter and producer ten plus because then they can count beyond 10 up into the 20's and 30's and another important thing here is that they can count unorganized collections. So they can count the number of lights on one of these chandeliers because they have enough mental capacity, they've made enough of the other, earlier skills automatic that they can have enough working memory to say, I got to figure out where I'm going to start and then go around and stop before I count that one again, right. Okay.

And it doesn't stop there either. Doesn't stop there. A lot of people think, oh, God. Okay. You said enough about counting. We should move onto arithmetic. Yes and no. The arithmetic skills start, the counting skills start coalescing with the arithmetic skills, but there's a lot of levels of counting still left. To do that, to talk about that, let's do some alphabet arithmetic. Okay. What this means is that I just want to put you in the position, kind of a young kid, and I can't because you all know this stuff already and have this automatized.

So let's just throw a little bit of monkey wrench at you. You can't use number words. You need to have a sequence because I want you to do it by counting, but instead of the number word sequence that we're used to, I could make you learn Japanese, ichi, ni, san, yon, roku, shichi, hachi, kyu, jyu, but that would take a while. So let's just use the alphabet. So if I ask you, count your fingers, you can't think one, two, three, four, five. You can't even go, what's the fifth letter? I know there's five, so I'll just count A, B, C, D, E, E. You have to count them, say, A, B, C, D, E. Then when I ask how many, you say E. Okay.

But we're going to do arithmetic, so like if I asked you what's B plus A, what is it?

AUDIENCE MEMBER: D.

AUDIENCE MEMBER: C.

CLEMENTS: B plus A? D? C? We're disagreeing on two plus one. B plus A. We have to start at B, right, and then we have to count up A, so we count up A and where do we land? C, C. All right. You got it? So B plus B is?

AUDIENCE MEMBER: D.

CLEMENTS: D. Then we're solid because we, but you can't say, I know two plus two, no, you can't do that. You have to start at B, say A more is C, B more is D. You with me? Okay. Ready? Now, at your table work out what's J plus E. Go ahead. J plus E, as in eagle.

All right. You all set? So don't tell me how you solve it. Let's solve it together to make sure we're on the same page. I saw a lot of people using their fingers. That might be okay if you count, if you enumerated them with the alphabet, right. So we start at J, A more is K, help me here because I stink at this. All right. B more is?

AUDIENCE MEMBER: L.

CLEMENTS: L. C more is?

AUDIENCE MEMBER: M.

CLEMENTS: D more is?

AUDIENCE MEMBER: N.

CLEMENTS: And E more is?

AUDIENCE MEMBER: O.

CLEMENTS: So the answer is?

AUDIENCE MEMBER: O.

CLEMENTS: O. All right. That's hard, right? That's hard. Now think about this. You've known the alphabet your whole life, right, from the age of five or something. You also already know how to count on because you do it with numbers. You're just switching symbol systems, and I saw people with their fingers up. I saw people writing. I saw people struggling, and I know some you were just there, ah, let her do it. It's hard. The little kids don't know how to solve it yet. They got to figure that out. They don't know the number sequences well or haven't had as much experience as you have with the alphabet, right.

And we expect them in first grade, come on, come on, come on, count out, how much is eight plus seven, right. And think about this, if I asked you, okay, now, let's try another one, what's Q minus K? I bet I can get most of you to say, I'll just sit here and wait, somebody else is, right? It's hard. It's cognitively very, what, let him do it? Is that?

AUDIENCE MEMBER: No. We were doing it together.

CLEMENTS: Oh, you're doing it together. Yeah, that's what I need to do because, listen, I've known the alphabet my whole life, right. And this gets back to counting. I've known the alphabet my whole life, but until I started playing this game, and I'm still not very good at it, I could only enter the alphabet at two places, A and L. So if you asked me what's three letters after R, I'd say, sure, L, M, N, O, P, Q, R, S, T, U. But if I don't get that running start from L, I just don't know what comes after R right away. Maybe you do.

My wife and colleague can do it up and down the alphabet any way. I don't know what kind of mind she has, but she can just go any letter and start going up and down. I can't, but I've known the alphabet my whole life. Again, we expect kids, you know the counting words, come on, what comes after 11? But it, to them, what's called an unbroken string. They can't break into it and start. They have to just start from this and, therefore, the very next level of counting here. Oh, I had this video first. Let me come back to that video.

The next level of counting is what Julie and call counter from N, N plus one, N minus one. Okay. That's the kid who now cannot just count up to 20 or 30, but can start at 15 and go forward or backward at least verbally. They can't always solve all the problems we're solving, but at least they, they're doing that. So what do we do with activities like that? We'll have a counting one, like a Nerf Wand or something. You have to have it soft because we sometimes give it to the kids to do, and they tap each other, and you really want a soft wand for that. They go wham, you know.

But we'll have a counting wand, and we normal just count the kids in the classroom, touch them once and only once, you know. Yes, that was my remote. I'll get it in a minute. But then we'll play other games with that such as like when you're touched with the counting wand, you start with one and keep counting. As soon as I take it away from you, stop counting, and then I'm going to touch somebody else, and they have to pick it up right there. Okay. So go.

AUDIENCE MEMBER: One, two, three, four.

AUDIENCE MEMBER: Five, six, seven.

AUDIENCE MEMBER: Eight, nine.

AUDIENCE MEMBER: Ten, 11.

CLEMENTS: So it gives everybody a running start, but they're not counting, so they still have to listen and think about that. Thanks so much. So simple activities like that and other activities really help kids with that skill. It doesn't stop there. Let's go back to that video. A four-year-old cannot believe what he can do. I know he looks bored. Okay. Here, I'll show it to you again. Okay. He looks bored. He likes math. I'll explain this. We got this assessment called a team, and it stops after kids get four wrong in a row, right.

And the assessor is supposed to know that if the kid is tired or goes more than 20 minutes, stop and pick it up the next day. Most kids, they're done in five or ten minutes. They reach their point and stuff like this. Forty-five minutes, these poor kids, and the assessor forgot to stop, so he's a little tired. Okay. I get that. But he perks up when things get interesting. She asks him, the assessor, to put out six objects. That's only the setup. Take a look.

[Videotape played]

CLEMENTS: How does he do that? Don't raise your hand. Talk to the people next to you. One more look.

[Videotape played]

CLEMENTS: What cognitive processes do you think he was using to figure that out? Talk to the people at your table and see what you think. So it's not too loud in here. So either you think this is pretty clear what he did or you're not sure. I'm not sure. We don't know. We just got the video too. I have my ideas, especially after seeing this like 50,000 times, but I could be right, I could be wrong. But what do you guys think? What were you talking about at the table? What do you think he did to get the answer? Go ahead.

AUDIENCE MEMBER: We said that he counted a block four of the blocks and then went five, six…

CLEMENTS: Ah, you think he, so how did he know two?

AUDIENCE MEMBER: Because he counted to four, and then he . . . four.

CLEMENTS: Right, but he said five, six. Where did the two come from cognitively, do you think?

AUDIENCE MEMBER: . . . two . . . it was just a combination.

CLEMENTS: Is it, now, you know, I originally thought, you might not mean this, but I originally thought, oh, he counted on his fingers. But actually he doesn't put up the two fingers until the very end as part of the answer.

AUDIENCE MEMBER: He could just be thinking that.

CLEMENTS: He could be thinking that. See, the two comes out right at the end, but he is doing this. I mean, you're absolutely right about that. So my question, again, is how does he come up with the two?

AUDIENCE MEMBER: Counted the counts.

CLEMENTS: Pardon?

AUDIENCE MEMBER: Counted the counts.

CLEMENTS: Counted the counts. I mean, it might have been that. You're skeptical. That's one possibility, but I think I don't think he counted the counts, but I think that's very possibility. What do you think? You look skeptical about that.

AUDIENCE MEMBER: Well, some first graders would probably say six…

CLEMENTS: Yeah.

AUDIENCE MEMBER: Instead of two, so I'm wondering how he figured out it was two and one.

CLEMENTS: Exactly. So if he didn't count the counts because first graders do say two. They know that they're trying to answer how many are missing, but they just said six. The last number word tells how many. Say six, but he didn't. He said two. How did he know? He might have counted the counts. You're absolutely right. We can't know. You're not wrong. I think there was probably something a little different. No, nobody?

AUDIENCE MEMBER: He could have subtracted.

CLEMENTS: He could have subtracted, which is even harder, right. Where was he looking when he said five, six?

AUDIENCE MEMBER: He looks up.

CLEMENTS: He looks up.

AUDIENCE MEMBER: . . .

CLEMENTS: I suspect, but don't know, he could say five is one, six is two just like if we were doing alphabet counting, right, that kind of thing. But he might have been looking and seeing five, six, kind of putting images in the air, and then reprocessing those images as two. That's what I think, but I don't know. You got to look at all his interviews or all the items, and you make your best guess. What does that mean for the learning trajectory? We talked about a counter for men.

The next level is where I think he is, a counter on using patterns. This is the kid who can count on four, five, six, but to do so he needs to use a pattern. Let's pretend he was adding rather than subtracting, right. If I asked him, what's two more than five? He might go five, six, seven. The six, seven pattern is what brought him to the seven. He didn't have to say, well, five, six is one more, seven is two. That gets really hard like you know from the alphabet stuff, right, keeping track of where you are in two sequences is hard.

But the rhythm, I get the rhythm, so a little teaching side note, what do we do about the rhythms? We do a lot, a lot of subitizing, which I'll get into more later. But it's kind of that instant recognition of number. We play number jump, where we flash a number like this, and then kids have to jump that many times and stuff like that, a lot of that. But we also play rhythmic subitizing. We tell the class, close your eyes [knocks three times], how many? You know, hold up your fingers or write a numeral if we're practicing writing or something to tell how many. Because if they have the rhythm of two and three and stuff like that, then they can use this as a way into counting on because the rhythm is numerical for them.

So there's subitizing that's visual, very important, there's subitizing that's rhythmic, also important, not quite as important, but still important. Where does it go from there? A kid who can count on, keeping track. Now we're hoping first graders can do this, and second graders should be really good at this, right. Counter on, keeping track where you say, because rhythmic falls apart, right. If I say 8 plus 7, and you're going to count on, 8, 9, 10, 11, 12, 13, 14, I'm not sure where I am now, right, but if I keep track on my fingers or somehow else, then I can put up 7, count those counts up and find out where I am.

Finally, second graders, we really wanted to counter forward and back. Those are the kids who say 37 plus 25, right, 37 plus 25, well, I'll go one 10 would be 47, two 10's would be 57, 5 more, then 57, 58, 59, 60, 61, 62. So they can count forward and backward in 10's or 1's and make sense of that, and that's where we need them to be. Okay. So where we go. Look at your learning trajectory for counting as a little guide.

It's on page what from our curriculum, so it's a high page number. It's like B something. Oh, it's not even duplicated. Okay. It's just three pages in, right. It says learning trajectory for counting. I'm going to show you some video. You're going to talk to the people at your table. We're going to do the best we can at trying to identify what level the kid's at. Are you ready? Here goes. I don't know what level these kids are at. I just grabbed a bunch of videos. So we might have to talk about this. Here we go.

[Videotape played]

CLEMENTS: If you can't pronounce the numbers right, it is a little more difficult actually. It's a really interesting phenomenon that, do you know what the two most often skipped numbers are? Anybody?

AUDIENCE MEMBER: Eleven …

CLEMENTS: Eleven might be. Thirteen and 15 come out a little higher than 11. Our daughter, Abby, when she was 3, counted 11, 12, 11, 12, firteen, firteen, firteen, sisteen. And then, a little later in her development, it was 11, 12, firteen, 16, 17, probably because they all sounded the same to her. And she was learning you don't repeat those number words, so she dropped a bunch that all sounded the same because she couldn't say F and TH differently, right. So it's an interesting phenomenon for a kid.

Anyway, I shouldn't talk too much because I'm going to ruin the memory of this for you. Talk to the people at your table. Think where might this kid be. You'd never really classify a kid on the basis of one experience, you know, but we're just taking our best guess at where the kid probably is. Go ahead. So don't worry about being wrong. And also nobody knows exactly watching one video anyway, so what, what do you think, and what's your justification for it? Who wants to be brave enough to start us off?

AUDIENCE MEMBER: . . .

CLEMENTS: Go ahead.

AUDIENCE MEMBER: Level nine.

CLEMENTS: Oh, I don't know the numbers. A publisher put in the numbers. I only know the names.

AUDIENCE MEMBER: . . .

CLEMENTS: Counter, counter producer. Why? Why were you thinking that?

AUDIENCE MEMBER: Because he's . . .

CLEMENTS: Okay. Just made a minor mistake. Possible. Anybody else, different opinion? Yeah.

AUDIENCE MEMBER: Four.

CLEMENTS: Whoa. Way down. Why? What's level four? Tell me.

AUDIENCE MEMBER: . . . all of the sudden he started going all over the place.

CLEMENTS: He started going all over the place, so level four is what?

AUDIENCE MEMBER: Reciter.

CLEMENTS: Reciter. So you wouldn't even give him corresponder, and I see that because he did start going like all over, so he doesn't seem to be keeping one-to-one correspondence, but she's arguing, well, yeah, but, so somebody, anybody come in between? Or just vote for one of the other or say in between? What would you say?

AUDIENCE MEMBER: Well, can I ask a question?

CLEMENTS: Yeah, you sure can.

AUDIENCE MEMBER: Are you allowed to use the counters as they count?

CLEMENTS: Oh.

AUDIENCE MEMBER: . . . keep them in the same . . .

CLEMENTS: No, no, no. That's a really good question because the very first strategy that works for kids and a good one to teach them, of course, is move the objects, right. These are glued down, so the kids, we purposely do that to see if they can do it in that kind of situation. Other items we have on the assessment? Of course. We lay them out in a, well, our early items are laid out in a line like you saw. Then later we lay them out in a bunch, and we see if the kid can organize them. And then they can move them, they can put them in a line, they can move them to a separate location. This one, no. This one, they're down, so he's stuck with trying to figure out a strategy for keeping his one-to-one correspondence. Does that change what you guys think about?

AUDIENCE MEMBER: . . .

CLEMENTS: What do you think about this level, though? Where would this kid be? Anybody else? I see both the points of the people. Both of them could be right based on what we saw, right. Because I've seen this kid do other tasks, he's about at one level before nine. Okay. He's not quite solid. He can't keep the one-to-one correspondence, but I would put him higher than the corresponder level or the reciter level for this reason.

On other tasks, so I'm sneaking, I'm bringing in information you didn't have, when things are in a line, he kept perfect one-to-one correspondence. Remember that the, one of the defining features of level nine, now here I go, you got me talking in numbers, and I hope I got the right numbers, of the counter and producer ten plus level is the ability to spatially organize. He's not quite there yet, but if you put him in a line, he'd be fine. It's only because they're disorganized that this kid loses it and gets confused. Okay.

And you'd have to see more video to see that. He definitely knows his number sequencer higher, he can keep one-to-one correspondence, but it's only in certain situations. Okay. Let's try another one. Let's try another one just for fun.

[Videotape played]

CLEMENTS: Sorry. So that was, obviously, not an assessment. It is an assessment, a, you know, a curricular-based assessment, but it's mostly instructional activity. We call it places scenes because he's got the farm animals and the place and he's supposed to put out some, count them, and then tell you a story that involves that number. It's more, it's instructional, but it's also an assessment. Where would you put him then? This is, I got these particularly, these are interesting ones because you could have a couple different levels based on what he just did. Do you need to see it again, or are you okay?

AUDIENCE MEMBER: I couldn't hear it.

CLEMENTS: You couldn't hear it.

AUDIENCE MEMBER: . . .

CLEMENTS: Oh, yeah, yeah. Okay. Let me talk about that because the background noise is tough on these. These are our homemade videos like I warned you before.

[Videotape played]

CLEMENTS: I'm muting this. He's, he counts and then starts retouching some and gets up to ten. Okay. His counting is in order. Then the second time he counts one, two, three, four, knocks it over, five, six with the same object, you know. So what, obviously, we've got to worry about is now, is he a counter of small numbers? Does he understand what he's doing? And it's only because they're mixed up and out of order and falling over, which doesn't help, that goofs him up? Or is way back to reciter, you know, kind of level. Talk to each other and say, what would you think? What do you think?

You know, one other interesting question, let's talk about this just for two minutes too. What task would you give him next to try to determine where he really is? Okay. So those two questions. Where is he, or what range of skills might he be at? What level, what assessment, how would we change this task to narrow down where we think he is? Go. What do you say? Who wants to start us off?

AUDIENCE MEMBER: . . .

CLEMENTS: Go ahead.

AUDIENCE MEMBER: Okay. We came up with the conclusion that we thought reciter ten, level four. He could count and touch for most . . . and then we thought the problem maybe with the things falling, the objects falling down that then put them in a line.

CLEMENTS: Yeah. Laying down in a line, right, so he didn't get distracted with that.

AUDIENCE MEMBER: . . .

CLEMENTS: Right, right, okay. Anybody, before I say anything, anybody else? Agree, disagree, say your own things?

AUDIENCE MEMBER: I would get rid of the . . . I would totally, for a child of that age and with the . . . issues, I would total take away . . . and give him something that he didn't have to . . .

CLEMENTS: Okay.

AUDIENCE MEMBER: So you really could get a clear picture of whether his math skill was . . . is everything upright and how it was placed on the map, just too many distracters.

CLEMENTS: Yeah.

AUDIENCE MEMBER: . . .

CLEMENTS: Okay. Good, good. Good one. And you wouldn't disagree with her to put them in a line or something probably too. Yeah. Anybody else? Because he might be down at reciter, but he might be up at corresponder, right. We just aren't really sure unless he's given the support he needs to be able to do that level. Yeah, did you have something?

AUDIENCE MEMBER: Well, did he give a total?

CLEMENTS: He did indeed, and he got it right. The trouble is, is that since she was guiding him so much and helping his hand get there, could he do it independently? It's a little ambiguous. And we don't have more video of this kid. This is it. This is a video a teacher took, you know, of her aid working with, so your guess is as good as good as mine, and we'd have to pursue that farther. But eight was correct. He finally counted it right when she guided him about that, but I doubt he's, that would mean he's like counter smaller numbers or counter up to ten.

AUDIENCE MEMBER: Like level six.

CLEMENTS: Yeah.

AUDIENCE MEMBER: Maybe go back and …

CLEMENTS: Exactly, exactly, right. But it's, now it might be just that the objects were falling down like she said, but it's doubtful that he's at the counter ten, counter and producer ten plus level, because there you can spatially organize your counting. And it's pretty sure he was fine going over and over objects, which that's going to develop in a next stage for him. Let's try one more. Back to this little girl. Remember she was for sure a counter of small numbers.

Do you remember? She counted five and aligned just fine. Let's take a look at this and say, well, fine, but is she higher than that? How does she do in another one of these objects glued down, right, situations with a lot of objects here. What would you say? You're supposed to start.

[Videotape played]

CLEMENTS: Are you kidding? Start over. I never want to do that again.

AUDIENCE MEMBER: . . .

CLEMENTS: Let's just talk about this together. What do you think? Can you picture a level that might probably be a good guess as to her? She proves she was a counter of small numbers before, so she can keep one-to-one correspondence in certain situations. What would you say about her work on this task? What do you think?

AUDIENCE MEMBER: . . . skip numbers, it throws off her correspondence.

CLEMENTS: It throws off her correspondence. She's skipping numbers. She seems, though, to be developing, right. The best thing I could say about her probably, see if you agree, is that she's almost at the counter and producer ten plus level. She clearly needs a little work on her number sequences, and she clearly needs a little work on keeping spatially organized, but that's a good instructional goal for her. It wouldn't be too far a reach to be working on those exact kind of skills. And the rest of the tasks we gave her prove that out.

So let's switch gears just for a second and talk about instructional activities then. We went through the counting sequence, not all the way to the end as you will see, right. There's a whole second column that takes us up to second grade as I illustrated before. But let's take a peek at some learning traject-, or the instructional activities. Way back for the kid at the reciter level, words are a song to sing. So we for sure do songs. We do, this kind of stuff we do, it's hard to find finger plays for pre-K, K people where the numbers go forward.

Everything's five little monkeys jumping on the bed, and they do down, right. That's great for counting backwards, so we invented some. And if you search for them, you can find them. Those are good. Those are good for reciter levels. We also do a lot of count and move kind of activities. Whoops. Went too far. You know, where kids will just march around the classroom, one, two, three, four. The emphasis is on the reciter level, but the stamping also lays the groundwork for one-to-one correspondence and things like that. So all those activities are good.

If I don't have it later, I want to tell one other thing we do is count moving patterns. We found that really effective, kindergarten and stuff like that, is to be counting one, two, three, four, five, six, so I'm getting a lot out of that. I'm getting the number sequence. I'm getting a rhythmic pattern of two, so you get rhythmic subitizing of two. And you're hearing two, four, six, eight louder than the other numbers, and so you start laying groundwork for even and odds and other things like that, right. Okay. What about correspondence level? Here's an example of an activity. We call it kitchen counter.

[Videotape played]

CLEMENTS: The kid doesn't have to know the cardinality principle, he doesn't even have to count out loud because the computer does that. But all the kids count with the computer, which is great with us. But they can concentrate on one-to-one correspondence, and if he clicks again on one he already clicked, it tells him, you clicked on that already. And if tries to move on without getting them all, it reminds him about that. Okay. So that's just very simple activities that focus on one conceptual level, one competence level.

We make number pizzas. We got all these games that are a pizza kind of theme. Kids have a count, they put three pizzas on there, and we always provide, and this is the important thing for learning trajectories, what if a kid's struggling there, what do you do? What if the kids can already do that? Move up to the next trajectory level. Don't waste kids' time when they don't need to be there. And then, finally, the number pizzas go up to the production level. Here's a little girl. Very first time she ever used this.

She's supposed to make a pizza on the left that matches the pizza on the right because twins want the same number of toppings on their pizza. She puts too many on.

[Videotape played]

CLEMENTS: Nothing succeeds like success. So and from there, we move on to now we don't give them a model. Now she has to do the same thing, produce, but you only produce given the oral, you know, three, you know, three, the number three pronounced and a numeral here, three as well pronounced. And we always let the kids play with the stuff afterwards. So they like playing and seeing how many pepperonis they can get on it. And they count way higher than they do in half the instructional activities from it. Okay.

And then I want to move onto the next level, which is production and all that kind of stuff, but show you an off-computer activity that emphasizes that although the learning trajectories tend to be kind of this linear sequence, the worst impression we could give is that every activity teaches this isolated little skill. Instead, activities should be rich enough to cover several trajectories and several levels for different kids. So here's a real simple game. Picture two kids playing, right, and so Jane and I are playing. I role a number cube. I get four, right, Jane.

And then I have to take the toppings out of a bowl and put them on a plate, not on my pizza yet. I got to put them on my plate. Because then I turn to Jane and say, Jane, did I get that right? Jane says, yes, and then I can put them on my pizza. Now it's Jane's turn. She's got her own pizza. She roles a number cube, and so the game goes. And if you want to have it competitive, whoever fills their pizza first wins. If you just want to play differently, everybody wins, and you just wait till everybody fills up their pizza, right.

[Videotape played]

CLEMENTS: Now that little boy had to push the number cube in his friend's face to get him to tell him he was right, but most of the time kids really like, although the teachers were worried that it would be too stilted, the kids really liked having somebody else, having something to do when it was somebody else's turn, and then that person would have something to do when it was their turn. The most important thing is, look at about the goals that this activity does. There's so many goals for this in number. Kids can make and imagine small collections non-verbally.

Now that kid said one when he rolled a one, but you could imagine a kid seeing the one, processing it non-verbally, and just grabbing one and putting it on his plate. Counting by ones to ten, you have to know the last word means how many when you count because then you have to count out or produce a collection. You might, with small numbers, subitize, quickly see how many are there and the like. And you, when Jane was telling me if I was right or not, she had to identify if two collections had the same number or if one had more. We keep doing this all year. Here are some kids at the end of the year very low resource community.

[Videotape played]

CLEMENTS: I love that little girl. She's going somewhere. She has confidence. I hope you could hear, even though it's our lousy classroom video, and the projector's kind of dark here, so it's hard to make out the details. But I hope you could hear. Look at the different solution strategies. Did you see some of the kids dump out their plate every time and just have to remake the next number. Other kids are yelling three plus three is six or the ubiquitous one more. They recognize right away, oh, I had six, seven, one more.

So counting on in a very limited, guided situation, but still in all developing those competencies, you know. It's within kids' capability and the like, but we need to have, to know how to do that. The good thing about, again, learning trajectories is you can differentiate instruction. There's, in the whole developed learning trajectories that we have like in our books and curriculum and other resources like that, the books that go pre-K through grade two are these two books.

The one on the left is more a research review. It goes one-to-one with the one on the right, and I'm not trying to get you to buy this so I make money. Just get it from a library or something if you want to, but the one on the right is more, it has one-to-one correspondence between the chapters. The green book on the left reviews the research, lays out the developmental sequence, the learning trajectory. The one on the right quickly summarizes that and spends most of its time on instruction and how do you teach it and what are good activities and the like.

So those interested mostly in that, those people working with teachers in professional development, the one on the right might be all you need for the teacher. That, it just goes into way more detail about this and presents kind of for each level if things go wrong, exactly what you do to generate the next level of thinking. Somebody who asked me, what do you do if things go wrong? These are the kind of suggestions for verbal counting, for object counting, for one-to-one or correspondence errors and for cardinality kind of things that research shows have been particularly effective for kids in developing those kind of skills.

And it goes up. The next page, sorry, was about skip counting and counting on and higher level counting kind of things. Sequencing is also really important in the counting process, and you guys were talking about that sequencing. We do everything with our little kids from building towers and building trains to on and off computer stuff to figuring out what is the missing number in a sequence and that. Incidentally, while we do our research, this is one area that higher than other areas differentiates ourself from the control groups.

The message to me is that most business as usual teachers are not doing very much with sequencing numbers and knowing what's more and less and stuff like that. We need to do more of that kind of activity.

[Videotape plays simultaneously.] Here the kids are just putting them in order. This is the beginning level. They have to grab the two, and right now they're supposed to put three there. The research shows that little kids like to help animated characters on screen. That's part of the research we use to try to design activities in which she's helping that little mouse get up to the . . . or whatever's up there, I can't remember. Here he goes.

He goes up there and . . . oh, no, God, I can't even see it. It's a mother cat gone to her kittens then, yeah, yeah. Okay. I'm not very good on biology apparently. So here we go. All right. Did you all write your numbers down? Let's do a few numbering activities, get you to stretch here because we're more than half way through. Ordering numerals. I don't have big numerals like that. I actually was supposed to bring them, and I looked in my suitcase and they're not there. So I don't know if I left them at the last city or what.

But I hope you wrote them down. I'll bet this table? Where did your table end? What's your highest number?

AUDIENCE MEMBER: Fifty-three.

CLEMENTS: Fifty-three. Okay. Fifty-four, 55, 56, 57. Okay. I didn't get to you guys, so I didn't tell you to write them ahead. All right. This might be chaos, but it's often chaos in class too. But let's give it a try. You ready? These are the kind of things you can do with these numerals. You can hang, actually let's just do this with the people just up to 20. Okay. If you're 1 through 20, I don't think I had anybody do 0, right.

So if you're 1 through 20, you might tell a story to little kids like the ducks are numbered, and they wandered all over, and now the mother duck has to get them in order, so. Or you're all garbage cans, and the wind blows you around. Or you're houses, and we got to get them in order. Any metaphor you want. So if you're 1 through 20, stand up, and we got to get you in a straight line over here. Go ahead. Try to get organized.

CLEMENTS: Somebody's taken control, right, making you do it left to right. Hold your number up in front of you so we can see. Cool. Now here's another, ordering is really important. There we go, number 15, we need you to make it a little darker next time. That's all right. Not now, not now. We'll have you in a minute. So you can also do, I won't have them do, I won't have them do, but you can also do this, you can have them put your right hand on the person one more than you.

Well, they've already, they've already kind of done that, so we got them organized. Ready? Here we go. Let's have, let's do it with just, no, let's do it with everybody. Everybody stand up and walk around and mix yourselves up like the garbage cans blowing around a little bit. Don't stay with people at your table. Okay. The slides that you can download off my website, the slides, if I flash by some of those things and you want to study it or get a PDF, this UBTriad.org has the, will have, will have, because I fixed some stuff, so I'm going to repost it, will have this presentation as of, what, tomorrow night or something like that.

You can download the PDF off it and look at those research-based kind of things. But thank you for bringing it up because I'd like to bring up one more thing. I mentioned subitizing real quickly, but I don't have any manipulatives. This was too big to have manipulatives on it. I need something, I don't see anything. Anybody got cubes or anything like that that? Dice, cubes. I need something. What do you got there for me?

AUDIENCE MEMBER: . . .

CLEMENTS: Like big cubes is what I'd use for this, but I can use anything right now for a second.

AUDIENCE MEMBER: . . .

CLEMENTS: Oh, four of them. Perfect, perfect. She's got something.

AUDIENCE MEMBER: I had them for lunch, and they told me I could take them.

CLEMENTS: No, no. This is going to stretch credibility as you see, but these are perfect for you guys seeing it, so that's great. And I think I'll put them up here.

AUDIENCE MEMBER: . . .

CLEMENTS: This eventually will get to Rose.

AUDIENCE MEMBER: Rosette.

CLEMENTS: Rosette. I was going to say Rosetta like the Stone, Rosette. Okay. Very good. This will eventually get back to your point a little bit. One more illustration. But it also illustrates a really important thing because I kind of went through subitizing fairly quickly, and it's so important to see how the learning trajectories cross fertilize each other. Because each one is kind of a linear process, although the levels are very rich. It's not teeny, step-by-step thing. They're broad levels of competence, but they also interact. Let me talk to you. Okay.

Picture somebody, oh, kindergarten or something like that where we're just teaching counting. A typical thing that teachers will do is just practice counting with their kids. They'll lay out some objects, probably not ones that roll, but I'm lucky to have these, right. Okay. So they might lay out objects and say kids count with me. So go ahead, count with me, let's go. Ready? One, two, three, four. From your perspective, they might know their teaching, reading skills, and they'll go left to right. Count with me. One, two, three, four, and then they'll put a different number up there and do that and count a few times and then, good, the lesson's over.

Here's how Julie and I teach that same kind of lesson in a more research-based way. Okay. We might say, listen, there's these balls, and I thought, you know what would be interesting is to find out how many I can hold in one hand at one time. So I went and grabbed them, and I really widened my hand and I get as many balls as I could. Will you count with me to show how many I can hold in my hand because later I want you to try it out. Okay.

And I would hold it behind my back, and I'd say, you know, actually if I had cubes, I'd hide them in my hand like this. And then we'd say, count with me. Ready? One, two, three, four. How many?

AUDIENCE MEMBER: Four.

CLEMENTS: I could hold four in my hand at one time. Now I put some balls out and during play time, you guys try it, see if you can hold four in your hand. I wonder if you can. You got smaller hands than I do, so I'm not sure you can do it. Try it later. And that's it, you know, what, less than 30 seconds. Talk to the people at your table. Maybe you agree, maybe you don't agree with us that this way is superior to the first way. But at least give it a try.

You know, suspend disbelief and say, okay, Julie and Doug think this is a better way. Why? What's different about this way than the other way that might convince us that this is a way better, 30-second lesson than the first one was? Talk to the people. See what you think. And not only that, that's the key. I mean, let's talk about other stuff that might be better, but that's the main point I was trying to get at. She nails it right on the head, which is kids know, kids can recognize one and two and sometimes three from two weeks of age in one way or another.

Okay. Picture this. Picture a little kid. Six months old is where they do this kind of thing, sitting in their mom's lap, looking at a screen. They flash four circles, sorry, three circles. Then they flash three diamonds in a triangular arrangement. Then they flash three of a different color square or a star, and they keep doing that. And the kid things this is pretty interesting. They're taking his pulse rate. They're measuring sweat on the kid's hand, and they're bouncing a harmless laser beam off the kid's eye to see exactly where they're looking.

The kid's looking at the screen. Heart rate is going. They get to the 25th slide of three objects, and the kid's looking around, and the heart rate goes down, and even for a six-month-old, boring by now. Okay. They show him the picture of four. Heart rate goes up, eyes back on the screen, start sweating a little bit. By six months of age, and other studies have found as early as two weeks of age, kids can differentiate between very small sets.

You know, even if the color changed, the shape changed, the arrangement changed. They know that the number changed. They don't know it consciously. They don't know that you added more. But somewhere in their neurological equipment, evolutionarily provided equipment, they know that the number has changed, that something has changed that's different, and they find that interesting. This ability, which later becomes subitizing, subitizing just comes from a Latin word that means instantly.

I see it. I don't have to count. I just know, right. We reserve subitizing if you can do it fast and if you can name it with a number. So the early recognition of the babies, the infants is not subitizing until it develops later. Kids with special needs, dyscalculia, often can't subitize any sets at all. It's just not there for them. Kids from, and it's very difficult, it takes a long time to develop, kids with, from low resource communities often are just really bad at subitizing.

You show them these, you say, what's here? They'll say balls. If you ask a kid from a higher resource community what's there, they say, four balls. There's a huge difference between those two. And, listen, I know a lot of you are like primary grades so you're less interested. But if anybody here is from early pre-school, like two- and three-year-olds and four-year-olds, I'll tell you Anula(?) from Finland closed the gap between the low resource and the high resource communities in number knowledge by doing one activity with teachers.

This is what she told them. She said, listen, you guys. One of the things I found is that kids aren't recognizing numbers spontaneously, so this is what I want you to do. I don't want you to tell the kids, clean up those water bottles. I want you to just say, could you guys clean up those two water bottles? I don't want you to say, line up at the door. I want you to say, listen, there's two doors, one here and one there. Could you just line up at, you know, the doors.

Look at these lights. Beautiful four lights in here, two here and two here. That's it. That's all they had to do. You didn't have to be forced, you didn't have to say, walking to school today, I took 32 steps. No. It's just spontaneously, normally naming very small sets closed the difference between the kids. Now it takes longer for kids who actually have a learning disability. But they need that because, and I finally got back to you, right? The biggest difference between these two is when we counted the first way, we said one, but you saw four. I said two, and you saw four. So what is it the first way?

It's just the teacher telling you, you might as well say, Bill, Marge. You might as well say A, B, C, D, right. But the second way, since I say I hid them on my hand although this is a real effort here, right, then when we said one, you saw one, when we said two, you saw two, and the subitizing combines with the counting to infuse the counting with cardinal meaning. Okay. Not instantly, not from this one trivial kind of thing, but it does illustrate the importance of cross-fertilization of the two learning trajectories. Subitizing provides cardinality to counting. Counting provides the ability to subitize greater sets.

Subitizing also, like we talked about with that little boy before, aids counting in sophisticated ways. What's eight plus three? Eight, 9, 10, 11. Rhythmic subitizing, supported counting to allow me to add. Okay. Both are important skills, foundational skills. What else is better? My Julie, wife and colleague, say about this activity then the first one. Go ahead.

JULIE SARAMA: We also got … extended… by challenging them do it themselves.

CLEMENTS: Yeah.

JULIE SARAMA: Because they'll definitely go try it.

CLEMENTS: That they'll definitely go try it. And you know what they're going to do. They're not going to have four. We told them in one hand. They're going to come up, look, I got ten. You didn't get ten. Well, good, they'll count higher, you know, because they'll cheat all they can to beat the teacher. So cool, but also it's more, it's a better activity because they're going to do more independently, of course. But it's also just way more motivating, right.

I want to do it myself. I don't care about you putting the things up there and forcing me to count them out. I want to do it, right. So it's more motivating for that reason.

AUDIENCE MEMBER: Also, to add to that, my students with the visual, they don't always get it. They need the concrete, so they need to put it in their hands.

CLEMENTS: They need to put it in their own hands. They need to iterate it themselves, count it out themselves and everything else. Absolutely, absolutely, especially special needs kids. Every, every possible sensory kind of thing and sensory experience is important. The other thing that kind of maybe I said it already, but just to say it in a different way. The first way, why are we counting it? The teacher said count it. The second way, why are we counting it? To find out how many. So I had them behind my back. You see nothing. And the first thing in your head is, how many has he got back there, you know. So right away counting is not something you do because the teacher tells you.

That's what happens to a lot of kids, right. I was going to put these down for myself. I really like these. A lot of kids, show Aunt Mable you can count, one, two, three, four. No quantitative meaning, right. So we want to make sure that we tell kids the reason we're counting is to find out how many. And then this is how we do it and this is what we say in the end and stuff like that. Okay. Enough on that. Enough on that.

Building up to arithmetic, and, boy, I got to hurry. I got to hurry. I got to, I'm going to skip through these fairly quickly. We build up through arithmetic. I'm not going to spend much time on this, about representations. Get to arithmetic and the primary grades real quick. Okay. Longitudinal study in grades one through three by the cognitively guided instruction group found that 90% of the kids use invented strategies whether or not they were in the classrooms of the half of the teachers doing traditional text book instruction or the half of the study where the teachers were taught about kids' thinking.

Didn't matter. Ninety percent of the kids in all those classrooms use their own invented strategies. It's ubiquitous. But the students using invented strategies before the teacher taught them the standard algorithm were better at base ten concepts, better at extending their knowledge to new situations. And it didn't matter. This group tended to do more of that, more teachers did that. But even if you were over in the text book group, if the teacher just allowed them to do their own thing before she or he introduced the standard algorithm, the kids had these advantages.

That's why this morning, if you were with me this morning, and I was saying develop, discuss, and use your own way of solving multi-digit problems first, it's a powerful technique.

[Videotape played]

CLEMENTS: How many people were with me this morning because you already saw these? I'm going to skip this actually and get on to some other things. Here we go. I don't think we got enough time. I, boy, boy, I talked and talked today. We don't have enough time to go into detail on this, but I'm going to just lead you through a couple just for fun. Okay. So turn to the arithmetic learning trajectory. That's what I'm doing now. It's beyond, there it is, adding and subtracting. There it is.

Developmental levels for adding and subtracting. I just want to illustrate a couple of these levels, just so you get the idea of what this does. You saw two of these videos, some of you this morning. Here's some more videos. We're going to say what kind of problem it is, problem type it is, and then we're going to say, is she using a direct modeling strategy? Is she using a counting strategy? Or is she using derived facts? Direct modeling, you model every number in the problem with an object.

Counting strategy you might model with some objects, but part of the time, you just use counting words. And derived facts is a little like doubles plus one. Ahh. I don't know seven plus eight, but I know seven plus seven. And so that's 14, and 1 more is 8. Okay. So this problem is Sam had, well, let, she reads it.

[Videotape played]

CLEMENTS: Okay. Sam had 11 stickers, gave 5 to his sisters, how many does he have left. Here she goes.

[Videotape played]

CLEMENTS: So is that direct modeling, counting, or derived facts? What would you say? She drew.

AUDIENCE MEMBER: . . .

CLEMENTS: She definitely counted, but we would call it, and I know I didn't give any introduction to this. We'd call it direct modeling because she had to produce some concrete representation, in this case, a drawing, but it's still concrete to her, of every item in the thing. A counting strategy is more like counting on or break apart to make tens that we talked about this morning. Okay.

So that's a direct modeling. Good strategy, but, you know, one that's great for early years, you want to move off that somewhere in first grade, okay, for most kids. We call the level find result, okay, if you're trying to link it up to the trajectory sheet.

[Videotape played]

CLEMENTS: All right. Now she used fingers, but did she represent every single number in the, in the problem with fingers?

AUDIENCE MEMBER: No.

CLEMENTS: No. She only used them to what? So our counting strategies, if you think back to that, would be counter on keeping track, right, right. Thus, it really is a counting strategy and our level on the sheet is called counting strategies, right, on the sheet that I gave you out, which I wish I could have spent a lot more time on this. Got too caught up with . . . let's try one more.

[Videotape played]

CLEMENTS: Talk to the people next to you, just 30 seconds. Is that a direct modeling? Is it a counting? Or is it a derived fact. Talk to the people next to you.

[Simultaneous discussion]

CLEMENTS: Oh, I'm sorry. Were you trying to look at the problem, or I moved ahead?

AUDIENCE MEMBER: . . .

CLEMENTS: Can you see it?

AUDIENCE MEMBER: The choices, correct?

CLEMENTS: Oh, yeah, yeah. Direct modeling, a counting strategy, or a derived fact? What do people say? Who's got an idea? It's a hard one, right, hey, because let me get you on a different tact while you're still thinking. I also tried to illustrate in my rather hurried way here at the end that we need to extend the problem types we give to kids. There are 12 problem types, right. We lay them out in the book, and I can give you other references to them, Carpenter and Fennema, and all those people from cognitively guided instruction.

All, most U.S. textbooks of the past, they're getting better and better, only did two types, the two types we saw first, separate result unknown. In other words, you had some, you ate some, how many do you have left? Or join. You had this many toys, you bought three more. How many in all? That's it. This is a compare problem where the difference is unknown. You can also have a compare problem where the first referent is unknown, right.

Rosette had five, no, no, no, Rosette had some toys, and Jane had seven toys. Jane had two more than Rosette. How many did Rosette have, right? So there's all this stuff. People, Russian and East Asian textbooks cover all 12 problem types by the end of first grade and then keep doing them every year thereafter. U.S. textbooks very frequently in the old days, they're getting better, did just 2 out of 12. No wonder kids don't have the schemas, the mental schemas for part-part-whole and figuring this stuff out.

Anyway, anybody got a guess as to what kind of strategy that was for that little girl? Don't want to say I'm not giving you enough. Hey. Here's another thing. Let me just say this to apologize for doing this. And if you're thinking, oh, my God, this is just too complex to use. Yeah, I mean, we work with our teachers, Julie, wife and colleague, and I in our research project, work with teachers seven full days on learning trajectories, full days. We get subs for them. They come and spend seven full days to learn the learning trajectories in year one, and five full days with coaching in between.

At the end of the second year, most teachers say, you know, I think I got those learning trajectories down, and I'm really using them. At first they're just a definition, a dictionary lookup where they are. Gradually during the second year, they become a powerful tool for formative assessment and individualizing instruction, but it takes a long time. No one can look at these and make perfect or good sense of these in two hours, especially when I'm slipping around from one trajectory to another. We never do that to the teachers either. We spend a day on one, right.

So I apologize for the brevity. I would call that one, just to move on, direct modeling. She had to directly model all the things one kid had and then all the others. Wasn't it interesting that she matched them up one-to-one? And I wonder if she ran out of paper or if she knew, had this way, because these are the ones that didn't match. But then also interesting, she had to physically separate those, and then she could cognize them. The, one of the characteristics of that direct modeling concrete strategy at the beginning is you have to visualize it.

You have to feel it, like you guys were saying, you have to see it, and then move them around. But you still can solve pretty interesting problems with your own solution strategy. Let's look at how a second grade teacher uses this kind of kids inventing strategies thing for multi-digit addition, subtraction.

[Videotape played]

CLEMENTS: Okay. Now I know I'm vastly outnumbered by women in here, but I got to be sympathetic to the guys sometimes because there's always a woman around to pull the rug out from under your confidence. So watch the little girl next to Steven here. Oh, sorry. Thanks. Watch the little girl next to Steven.

[Videotape played]

CLEMENTS: Okay. There's more there. But I hope you get the basic idea that, you know, these kids are at the, beyond the stage of counting strategies. They're moving from counting strategies to derived facts to composition of number and stuff like that. But they're inventing their own ways first. Now in this particular class from Connie Kamii, who made the video with a teacher friend, really believes never teach the standard algorithm, never. You know, kids just figure it out and do their own strategies.

I tend to think, no, no, no. The research is pretty clear. Do this first, but connecting it then to the standard algorithm does nothing but give kids another good technique, you know. The error comes when you teach them carry the one, and they don't understand it, right. I mean, my son came home in second grade, said I know how to add multi-digit numbers. And I said, great, Ryan, tell me more about that. He said, well, it's dating. Dating? I said, what do you mean, dating?

And he says, well, you know, if you got eight plus seven and then it's like eight plus seven, go on a date, and then you add that up. And he counts on fingers, that's 15. And then when it's time to go home, the five goes to his home, and the one goes to his home over here, and then, and he said, but I've got some questions. And I said, yeah, I got quite a few myself. And he said, well, Dad, but what's your question first? And I said, he said, Dad, what's dating? So and not only was it a non-mathematical way the teacher was trying to teach him, but it was a metaphor that didn't make any sense to him any way.

What do we do when we want them to go to, oh that beautiful research that was done with that. Let me tell you real quick because I'm almost out of time. They gave, the traditional taught group from a textbook scored 23% on a regroup, could explain regrouping. Eighty-three percent from the classroom you just saw could explain regrouping, right. They asked the kids, there are 26 sheep and 10 goats on a ship, how old is the captain? A hundred percent of the traditional kids said 36. Only a couple kids in the other classroom, the classroom you saw, said that kind of thing, right, 27% did.

So but I want to move on from this and just get to, there we go, what do we do when we want to connect the kids' thinking then to the standard algorithms because that's really going to be important? There's nothing wrong with doing the kind of thing like here where kids write all the totals. And did you see that every single kid in that classroom, and other research verifies that, prefer to start with the big numbers, not the ones, right.

When we're doing, talking about the supermarket shoppers this morning, they all do the bigger numbers first, the highest place value. So doing this kind of writing all totals works. If once you get closer to the algorithm, here's something that looks almost exactly like the standard algorithm, but research shows it's way better for kids for two important reasons. All you do is you add the 9 and the 7, you get 16, and you write 16, not put down the 6 and carry the 1. You record 16, but you make the 1 just smaller right here, but you write 16. That does two important things for kids.

The profound thing it does, is it maintains 16 as an entity. The less profound, but still important thing is, if you put the quote upon carry up here, kids have to add the carry to the eight. That's how they're taught. They get nine, then they have to add the nine to the five, but there's no nine there. So a lot of kids forget, go back to the eight, add the eight and the five, and they're off by ten, right. Here you add the eight and the five. You get 13, and it's easy to add one more ten, so then you get the fourteen 10's. You do the same thing. You put fourteen 10's and you proceed. Okay.

Little teeny difference, but holy mackerel, a fantastic difference for kids, especially kids who are struggling with mathematics, those kind of things. Now some kids should do expanded notation, some kids have to do all kinds of things, kids who really suffer from a learning disability in mathematics, but those really help. Okay. And for less advanced kids, you know, this kind of expanded notation can help them keeping in terms of place value.

How about like subtraction? Let's just do that real, ooh, quick. Here's one thing that for kids struggling in mathematics, okay, whether learning disabled or just having mathematical difficulties, this is the strategy to do with them if and when you get to the algorithm stage, right, which is ungroup everywhere first, then subtract everywhere.

The reason most kids screw up on the subtraction thing is they, you know, they regroup and then they subtract, and then they regroup even if they don't have to because they're flip-flopping back and forth. It works just as well and just as efficiently to do all the regrouping first and then go ahead, place value at a time, and do the subtraction. Okay. So you concentrate on where you have to regroup. You do it all before you start subtracting, and then you do the subtraction. Okay. Got to stop.

So I'm just going to go, just go to the end. I can't do the geometry I had, oops. Oh, God.

AUDIENCE MEMBER: Do you like TouchMath?

CLEMENTS: Yeah, exactly. It's okay. It doesn't, the research is, it's fine. And I'll talk to you about it later. Let me sum this up. There's no research that proves it's way superior. I would like to tell you guys just one more thing. We also have a website that really has helped our teachers figure out learning trajectories because what it does is kids, people can look at the instruction, and then they can click on the related development, and then see videos of the kids at that level of development, and then they can slide up and down the levels of development and compare kids at different levels to try to figure out what makes sense for the kids at various levels.

Takes time to use learning trajectories. Good. I only have 30 seconds, and this will take less than that. I love this teacher who said, you know, I was sitting with a kid preparing for parent reports, and she was able to do verbal counting to eight. And then when she slowed down, she could get to 11. So I said, can you make me a group of six, and she did. Then I added, I think I did 12, and she couldn't do that. And then I noted, now I'm thinking in the trajectories. She's on her way to being a counter ten. She's a counter of small numbers. She's in between the two. That's what I was thinking as I did this.

When teachers can internalize these learning trajectories and use it to plan instruction for kids, it's a powerful tool. It takes way more time than two hours, and I apologize for going so quickly with you guys and stuff like that. But we think, Julie and I, wife and colleague, believe that that's a really powerful way to get kids going. It's 2:30. Thank you for your time. Have a great rest of the conference. I'll be here for a couple minutes afterwards.