Frameweld

PaTTAN 2011 PDE Conference

Math Lessons from Research

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MUIR: My name is Allen Muir. I'm from PaTTAN Harrisburg, and I'm here to introduce Dr. Clements. He's widely regarded as the major scholar in the field of early childhood mathematics education. At the national level, his contributions have led to the development of new mathematics curricula, teaching approaches, teaching training initiatives, and models of scaling-up interventions, as well as having a tremendous impact on educational planning and policy, particularly in the area of mathematical literacy and access.

He served on the president's National Mathematics Advisory Panel, the National Research Council's Committee on Early Mathematics, the National Council of Teachers of Mathematics National Curriculum, and Principles and Standards Committees, and is a co-author of each of their reports. He is presently serving on the Common Core Committee of National Governors' Association and the Council of Chief State School Officers, helping to write national academic standards.

A prolific and widely-cited scholar, he has earned external grant support totaling nearly $19 million, including major grants from the National Science Foundation, the National Institute of Health, and the Institute of Educational Sciences of the U.S. Department of Education. Let's give Dr. Clements a warm Pennsylvania welcome.

CLEMENTS: Thank you, Allen. Appreciate it. And by widely, widely-known as a major scholar, you know we mean my wife and my mom and, you know, and I have letters from them in case, in case, you know, you don't believe it. All right. I think I'm preaching to the choir with this beginning one, but let's get on the same page as I share with you the fact that we really need better mathematics. Can you guys see the screen okay? The only controls seem to be to really put this side in the dark. But if ever you need it, you just signal that in some way, okay?

Let me show you a few actual advertisements that I've gleaned off the Internet and off of newspapers that seem to indicate we really do need some better mathematics, right? Get 50% off or half price, whichever is less. Great, great. And this one really brings them into the stores in western New York. Okay. Buy three for the price of three. All right. There's a deal we can't pass up. And, finally, a lot of you work with older children that I've been working with lately. And, you know, statistics can be a real problem.

Well, it's apparently a problem for writers too because they claim that statistics show that teen pregnancy drops off significantly after age 25. I can see why that would be true, right? We really do need better math. You guys are doing well, though. I wanted to make you feel good about yourselves this morning because you know the math, you get math. And, interestingly, look at this, the family wealth of people that know mathematics is on average, when they retire is $1.7 million compared to people who don't know math well, who are down there at $200,000 when they retire. Huge difference, right? And we're not talking about that you have to know calculus, right? They're fairly simple word problems they did in this study illustrated here.

If the chance of getting a disease is 10%, how many people out of 1,000 might be expected to get the disease? That's all you had to do to be here. It's really important then that not only we feel good that we'll have more than $1.7 million when retire, but that we help many more people have at least the kind of skills that are necessary for this kind of thing. And kids can do so much more. Let me tell you a story about Key. True story. Okay. He's a third grader in western Connecticut, and his teacher was reintroducing at the beginning of the year subtraction with borrowing.

So she puts the problem on the board, 68 take away 24. And she starts. Now you can't take eight from four. Well, right away Key interrupts her and says, well, yes, you can. Eight from 4 is negative 4, and 20 from 60 is 40 and negative, woops, sorry, negative 4 and 40 is 36. Now it's probable that no one had taught Key this strategy or this algorithm. It's probable no one in western Connecticut had ever done subtraction exactly like that. It was an original creation of a third grade boy.

The potential is there. We don't realize it a lot, but, again, it's increasingly important to get everybody knowing mathematics. And everyday life, like the retirement thing would say, to meet the needs of the global economy and educationally too. You might have some colleagues in science education who argue all the time. They're even more diverse than we are, because the physicists all say, you know, in the end, physics can explain everything.

So people should take more physics in high school. That underlies all science. Biologists say it's evolutionary theory, really, that underlies science. That's what you should take more in high school. Chemists say the same thing. So here's a little study that people did. They just collected how many courses in each of these fields, biology, chemistry, and physics kids took in high school, and then how did that correlate with their grades in those same three subjects in college. So here's what you find.

If you take more courses in high school in biology, you do a little better in biology. You know, the typical confidence interval, so it was statistically significant, but it really didn't matter if you took chemistry or physics. Similarly, okay, high school chemistry, health chemistry, nah, and physics, although not significant, seemed to have a dampening effect a little bit, right? And college physics helped your physics, but not the other two subjects. So none of them won.

It seems like uninteresting study, you take more of subject A, you do better in subject A in college. I mean, what's, you know, this is the kind of thing that taxpayers roll their eyes and says, how much money did you spend to find that out? Except that black bar, right? Because the more math you took, the better you did in biology, chemistry, and physics. And in fact, some of them, although not statistically significant, seem to be more important if you took math than the subject itself.

Math is increasingly important in those, and people agree. This is very recent research that came out of public opinion kind of surveys that they do at Michigan State University. These are critical public school objectives that merit financing, so they're asking parents and people who aren't parents. I mean, they're asking just the public at large, what are the most important things we should fund in education? Look at what's number one. Seventy-four percent of the people said, we need standards in mathematics and science.

Critical, math is a critical subject for both parents and children. They both rated it the most important school subject. Both see it as a subject you need to get a good job. Both see it as a subject they'll use the most. But kids lose confidence in it too. The same survey shows that most kids like school and math. Seven out of ten like math. Only 55%, however, think they're good at it. And all those figures start to decline, especially at seventh and eighth grade.

Parents might be feeding into that, because they also lose confidence in what their kids know in math about the same time. And a lot of parents, and pass this on to their kids, believe that certain children will just never get math, that it's an attribute, it's a personality attribute or cognitive attribute. Thirty-nine percent say it's okay for a kid to say, I'm not good at math. And twenty-nine percent think it's hereditary. You're either good at math or you're not, and it's kind of predetermined for you. We need opportunities for all those kids. We need to convince people it's about working at it and learning it, not about having the right genes.

And we also need to serve the other end of the spectrum. Ninety-one percent find that our schools need better ways to ensure that the kids who are the brightest kids have these kind of opportunities. So this morning for the rest of the morning, I'm going to be talking about lessons that I think we've gleaned from the research. Our own research results, which are published in these two books that are underneath here, here and here.

And then the reports that I've been part of that in the introduction you heard about, a National Research Council report for the National Academy of Sciences, the Foundations for Success, which is our report from our Committee on the National Mathematics Advisory Panel, and then the curriculum focal points by NCTM, all of which I was a part of. So I'm gleaning some stuff from all of those to try to present what I think are some of the most important things for mathematics education, especially to meet the needs of kids with special needs.

So any time you read an article that says how to present, they always say people can only take away three to four major points. Just give three to four major points. You can't take away any more than that. I couldn't boil it down. Okay. I got like twice as many as that. All right. So here's what I'm going, just let's negotiate here. I'll present like eight. You memorize three or four. All right. You can pick the three or four. You hate three of them. Great. Toss those out. Try to take three of four of them away.

One of them is the gaps are striking. The next one is less is more. The next, we need to use truly research-based education. Everybody claims that their stuff is research-based. I want to talk about what it really has to do to be research-based. We need to connect informal and school mathematics, especially the younger the kid is and especially kids with special needs, but for everybody. We need to include geometry in a more substantive way in curricula.

We need to meet the needs of all students. There's hundred of ways we could talk about doing that. I'm going to stress technology and some particular research findings on kids with special needs. And, finally, we need to use learning trajectories, which, if it's a notion that's not familiar to you, I'll be dealing with that a couple times through the talk. Okay. So once again, we know that school mathematics in the United States is not working well enough for enough students, but I wonder if you've seen this.

You know, the United States, I know it's probably too hard to read the countries in the back. U.S. is here towards the bottom. Other countries tend to do better than us. We don't outperform most of the high performing countries and the like, but have you ever thought of this? Where are the kids who have the most opportunities to learn math in the United States? With teachers such as yourselves, right? Where do they fall? They fall, there's the U.S. I circled that for you in the back. They fall up there, second in the world. If you find the kids who have the resources to learn mathematics education, they do remarkably well.

The kids at the other end, not so well. Right above Nigeria and Swaziland. Okay. Because there's a ten-to-one difference in what we spend on the kids at the top and the kids at the bottom on their mathematics education, ten-to-one difference. Recent PISA results just came out like a week ago, right, so I hurriedly put this together because Arne Duncan says this is a wake-up call for the United States. The U.S. score is 500. The average of all the countries is 493. We're in the middle of these countries. We're nowhere near the top.

But let's take a look at different U.S. schools again, right. Here are the PISA results. The average, as I said, was 493. The U.S. average is 500. Look at the difference between the results of the schools that are in the top, that have fewer than 10% of kids who are below the poverty line compared to schools with different percentages below the poverty line. Look at the range there. And even if you look at this, here are schools, U.S. schools with less than 10% of their kids in poverty. Look at how those schools compare to the top performing countries.

How about those, like Canada, that have, Canada's 13.6% poverty. When we group those schools together, we score okay. And, again, on down the line for greater and greater poverty. Let me just go to the last one. Here's Mexico, and we're scoring about the same. It's about opportunities to learn and funding and resources for those opportunities to learn.

So my first lesson is gaps are striking, and we need to be really, we need to give a lot of attention to those kind of gaps and do what we can do. Because these are the kids that fill the Tier 2 and the Tier 3 levels, and prevention and intervention for those kids is just so important. Another lesson from those same comparisons is that we need to do less is more. Let me lead to that a little bit by reminding you of some TIMSS report. This is hard to see.

I'm sorry for the people here. If it gets too dark for you and stuff like that, I'll flip this back up, but this will be a little better for people. Here are the percentage of the average German eighth grade teacher in the TIMSS teaching study. The percentage of time that those teachers spent on developing the meaning of concepts, that's the green, versus just stating a concept or giving a definition. Okay. So a little less than three-fourths of the time, they're developing kids' understanding of those concepts.

Japanese teachers a little more than three-fourths of the time, a little less time just stating the definition. United States was the only country at eighth grade that flip-flopped those two percentages, spend a vast amount of time giving a definition or having kids reread a definition, not developing the conceptual meaning of these things.

And that's correlated with those same teachers, the same study. In Germany, they spent 23% of their time on lessons that mathematicians categorized as having high-quality mathematical reasoning. Medium quality and low quality was about 20%, 40%, excuse me. Japanese, higher, 57% medium quality up to 30% high-quality mathematics reasoning in the activity. And United States, 0% high quality. Vast majority of it, again, stating definitions and stuff, which would lead to low-quality information.

Not people like yourselves that come to conferences, but across the country, there's a lot of work we have to do with those teachers. So because those high performing countries tend to really focus in on the most important mathematics and do high-quality reasoning, that's why we at the National Math Advisory Panel said that the mathematics curriculum in grades pre-K through eight should be streamlined and emphasize a well-defined set of the most critical topics in the early grades.

Our curriculum are huge, especially pre-K through eight. We need to spend sustained time on fewer key concepts. So I say, less is more. Not less mathematics, fewer topics and more concentrated time on those topics is the most important lesson we've had. Less but what, you know? The National Math Panel results suggest some topics. I want to give you a little caveat about the National Math Panel report. Our charge was not to say what topics are important to teach. They were what topics are prerequisite for learning algebra.

So the only geometry and measurement that's in there is they have to directly connect it to learning algebra. So you got to be careful if you do look at those or think about those that you realize that. On the other hand, the curriculum focal points were designed to say, what are the three big conceptual ideas pre-K through eight that we should look at? Because this is what we looked at. Barbara Reys and her group studied standards from around the state when we started the curriculum focal points.

Look at the span of how many learning expectations kids faced at grade four. From California, who had 43, I don't have your wonderful state on here. Texas had 32. Ohio, 48. And look at Florida, 89 learning expectations. God forbid, they have a few days off the school for a snow day or something like that, right? One hundred eighty days of school, 90 learning expectations. You miss two days of school, you just lost a learning expectation. Every two days, you have to meet a learning expectation, right?

This is one of the reasons we said, we need a Common Core, which I'll get to in a minute, our curriculum focal points. And I'm going to have you take a look at those in a minute to get just a break from me. You've got the curriculum focal points pre-K through eight, I hope, right here as well as, oh, I hope it's obvious that I didn't just put the pictures out here to be pretty, right? The URLs are the important thing, in case you want to look something up that I've mentioned.

One thing I wanted to tell you about before I let you dive into that just for a second, right, is don't miss some of the important stuff. I'm mentioning learning trajectories, and actually, although we limited ourselves to one page per grade level, there's a little hint of the learning trajectories even within grades and definitely across grades, so look for that. Also, there's some things you might not notice on a quick reading. At second grade, for instance, a quick recall of addition and subtraction and then developing addition and subtraction multi-digit skills. Okay.

Notice this paragraph. Let met blow it up for you so even people in the back can read this. It says, children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multi-digits, whole numbers. They develop fluency with efficient procedures, including but not limited to standard algorithms for adding and subtracting whole numbers. Understand why the procedures work on the basis of place value in the property of arithmetic operations and use them to solve problems.

The key words, develop, discuss, and use. When we started this work, there were still, we were kind of at the height of kind of math wars kind of a thing. And conservative mathematicians were saying, you got to know your basic facts and learn the standard algorithm. And on the other side were people that said, people like Key, do wonderful thinking and reasoning and working out strategies on their own, and that's a better way to approach it. Seems opposing point.

Rather than a week's negotiation between the two and a compromise, we came up with what I think is a very powerful synthesis of those two, because the research is not against kids knowing the standard algorithm. What it's against is the standard algorithm being imposed on kids before they've constructed their own understanding of things. So we say, kids first should develop, discuss, and use. Develop, invent their own strategies, discuss those, talk about why they work, how they work, why I think mine is good. How about yours? You defend yours.

And then, finally, use those to solve problems and then develop knowledge of efficient procedures, including the standard algorithm. It's a powerful synthesis of what a lot of people saw as opposing positions, and you'll see the same language in grade two. Just look up here in multiplication facts and stuff like that. Develop and discuss and use and the like. Okay.

So before I go to the Common Core, let's just take a minute in case you want to just check something out in there to look at the grade level or levels you're interested and talk to people around you. I'll give you just a couple minutes to peek at those and familiarize yourself with that. Go ahead. Okay. Let's move on to the Common Core, which is thick enough that that was kind of impossible for us to duplicate, but I certainly hope you look at that.

Because it used the curriculum focal points, that's what we used when we started the Common Core kind of process. One thing I want to point out. Bridging the curriculum focal points that you have in the Common Core that we did is this notion of process or practices. So in the curriculum focal points, you'll recall that the top of every one says that these are the content emphases. It's essential that those focal points be addressed in the context to promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Likewise, the Common Core starts out with the, when you read the standards, those are the content things, you should always make sure kids are engaged in those mathematical practices, right? And they're different, they're phrased differently, but they include some other ones like attending to precision and the like. But the idea of looking for structure and reasoning and everything is right there.

So let's get into process just a little bit with a problem for you. This is like first, second grade problem. No problem for everybody here, right, but maybe a little bit different than the kind of problems you're used to solving. But it speaks to this process thing, and I'll tell you more about that after we solve it. So work with the people next to you. I went to first grade teacher and to first grade. I've been doing these kinds of problems all year, but I give the kids this.

I say, listen, I was out on the playground the other day. And it was really interesting, because all of the sudden start talking about their sisters. I mean, all of a sudden, people started saying, well, who's your sister? She's my sister. She's my sister. She's my sister. And they were all pointing. I thought it was so interesting. I grabbed a piece of paper. I couldn't draw the kids, so I just made little dots for all the kids on the playground, and then I just used an arrow.

So if one child pointed to another child, I drew an arrow from the first one who was pointing to that person's sister. So this child was pointing to this child and saying, she's my sister. Can you see the direction of the arrows in back? Okay. Super. So I wrote down all the kids as dots and all seven arrows. All, one, two, oh, geez, I know there were seven, seven kids were pointing to another kid and saying, she's my sister. There were seven arrows when I was, I forgot one. Can you guys help me? There's one arrow missing on there, just one. Can you figure out which one it is?

Go ahead. Talk to each other at the table, think about it, and try to find which one's the missing arrow. Go ahead. Well, it was a boy? What's this? Girl or boy? How do you know?

ALL: She's a sister.

CLEMENTS: She's a sister. Okay. How about this one?

ALL: Don't know.

CLEMENTS: Don't know, don't know. How about this one?

ALL: Girl. Girl.

ALL: Boy.

CLEMENTS: Boy, maybe, right, unless, what, that's the missing arrow. I don't know. Give you 30 more seconds. Go. If you don't have the answer, keep talking.

No. No blended families. No bisexual kids. We don't, just keep it straightforward, everybody. Yeah. Thank gosh I don't have a diagram with that family up there. Okay. All right. Who thinks they know? Who, what, anybody confident? Go ahead. Nice and loud for us over there.

WOMAN: … bottom.

CLEMENTS: Okay. Which bottom, where bottom?

WOMAN: Okay. The third one to the left.

CLEMENTS: Okay. Right here.

WOMAN: Yes. I think there needs to be an arrow from that child over to the two sisters . . .

CLEMENTS: Why?

WOMAN: Well, because if they're sisters . . .

CLEMENTS: These two.

WOMAN: They're all in the same family, have the same two parents. If the two girls there are sisters and even if that's, you know, that boy whose sister is the top sister, girl . . . he's also going to be sisters with the other girl.

CLEMENTS: All right. Anybody disagree? Who had the same basic idea? Lots of people. Let's see, the official answer. Yes, very good. You get one more Mr. Goodbar. There you go, there you go. Whoa, whoa, whoa. Okay. Very good. Now here's the point, right? Just first of all, it's just a break in mathematical reasoning, but think of, I heard you guys talking, we heard reasoning, we heard connections to what you know about families and all that kind of stuff.

Reasoning, problem solving, communication, all these are mathematical processes. But my main point here is I love to take this to my mathematician friends, who, after they solve it, they say, you know, everybody thinks math is about numbers and arithmetic, but this is closer to what I do every day than adding a column of figures is. This is the mathematics that I do, figuring stuff out and justifying it, right? So the process is really important, whether you do it this way or whether you do it in the context of other number and geometry work.

Okay. Back to the Common Core. There's two things I'd like to mention. I hope you read the Common Core. There's two things that you may have missed that I want to make sure you don't miss when you do read the Common Core. Okay? First of all, look at what it says at the beginning of second grade. Kids develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences. Exactly the language from the curriculum focal points.

So this is a marvelous second example of how people on these so-called math wars, the second side to those, got together and said, how can we synthesize those to get the best out of both worlds? And the consistency between this and the curriculum focal points can't be more clear. What else might be missed? That learning trajectories, called progressions by many people, learning progressions, and you hear all different kind of things.

We're not even calling them learning progressions any more. We're calling them mathematical progressions because so many, when you get into high school, there's just not a lot of research on the psychology of learning these things. It's much thicker the lower you go in ages. But in one way or another, they're learning trajectories. They're based on this notion of what levels of thinking do kids pass through on their way to mastering some piece of mathematic are at the heart of the Common Core.

When we started the Common Core, we didn't start writing standards and saying, what should be in second grade? We started by writing learning trajectory. We wrote developmental progressions of levels of thinking first. We passed those around, agreed, disagreed, argued and stuff. Then all those went to three writers who, in a rush had abstracted from those, then what goes in kindergarten, what goes in first grade, what goes in second grade, and all the way up. Okay.

Now the plan was to publish those learning trajectories as an appendix to the standards. Didn't happen. Why not? It took us way longer than we thought. As usual, the governors were pushing us. We need this now. But the good news is we're going to have those pretty soon. I'll return to that. And if you look at the flow of the different topics in there, you can already see there are these kind of concentrations that lead up. For instance, here's some learning trajectories for various topics that lead up to algebra.

Projects still under way is that we're writing, re-writing the standards of progression. Because they were written first and then modified and modified and then turned into standards, they no longer fit exactly the standards. So right now I'm working on geometry and measurement. Other people are working on place value number, algebra and everything else and high school stuff.

And we will be publishing on the website what was initially supposed to be the appendix with it, the developmental or learning progressions that underlie the standards, so people can see the continuity of kids' thinking and teaching that are under there. So look for that, because that's like, to me, the most important piece, especially for the kind of work you guys do with teachers and the like. We're going to try to develop better sample tasks as well.

We're hopefully coming out with a kind of technical manual that will help provide some details, and then we don't know. We're searching for funding to try to get some professional development resources out there too. That's a question. The other ones we're doing, but that one we're just looking for resources. So any questions on the Common Core that come up, please ask me. Any questions you have about anything as I'm talking, just wave me down.

Let's move on, though, if not. We talked about then gaps are striking, less is more, more focus on key concepts, and then, finally, I'm moving to the next one, research-based education. We really got to, especially kids with special needs, you need to be effective and efficient, right? So everybody claims their stuff is research-based, but not a lot of people have respect for research.

Gilbert says I didn't have any accurate numbers, so I made this one up. He says, studies have shown that accurate numbers aren't any more useful than ones you make up. He's asked, how many studies showed that? Eighty-seven. So there you go. Everybody claims their stuff is research-based, and there's lots of studies that back them up. How do we know? Well, in the Building Blocks project, this is an early childhood project that I'm mostly talking about this at 12:30 in the afternoon.

But I, right now, I want to give you the idea that, what did, we came up, we had to think up a framework for if we were going to claim we were research-based, what would we say? And we wanted to claim that. And so what we did is said, there's three broad categories of research that we're going to lead it through, ten phases. And then we believe we could truly be research-based. The first phase, the a priori foundation, is just like, review the research before the fact. Before you write a curriculum, know what the heck the research says.

So, for instance, you'd have to know about what's our basic approach. Our basic approach for Building Blocks is finding the mathematics in and developing the mathematics from kids' every day activity. It's preschool through grade two, so that was real important to us. Subject matter. What are you doing that's the most important topics? Well, at that time, we didn't have the Common Core, but we did have the curriculum focal points, which I was just writing at the time, but we had that.

And we had some, a conference that Julie and I had been funded by National Science Foundation and Exxon Mobil Foundation to give that got a lot of people from the different departments of education and researchers together. So we thought we had a firm foundation on what was the most important mathematic. Then look at pedagogy. So Building Blocks included, it's a minor part of the curriculum, but an important, small part, technology. So we wanted to look at what does research say about what makes technology particularly useful and effective for young kids.

And then you spend most of your time, the huge middle of the time developing, testing, and ensuring the efficacy of these learning trajectories, these levels of thinking through, which kids pass on their way to developing a mathematical competence, and correlated instructional activities that are exactly matched to those levels that are designed to move kids up. More on that later. Then you go to the big evaluation, but you spend most of your time in formative evaluation.

Julie and I taught the activities ourselves to just one or two kids, then do a small group of kids. Then we work with one of the 35 teachers that helped us develop this curriculum in their classrooms. They knew the stuff. They had helped work on the stuff. And then, finally, we gave it to teachers who didn't know the stuff to see if the support materials were adequate for those teachers. And only then did we go with, to what people call the gold standard, randomly assigned classes to the controller or the new curriculum, see which one does better.

And you only do this in four to ten classes at first because it's a waste of time and money if the materials don't work. And in fact, the most important part of this research framework is every stage has to proceed or you stop or you go backwards and fix and go on. Okay. And then only then do you go to the large-scale . . . and that we did. I'll burst out, it would take weeks to talk about all the qualitative research in the first eight phases.

But here we are with four classrooms randomly assigned. This is the number. Here are the kids in the Building Blocks group. The control group gained a lot. They doubled their scores, but we doubled what they, their gain. In geometry, it was even a greater effect size. It's more than two standard deviations difference, probably because early childhood teachers think I showed them a rectangle and a circle, what else is there to geometry? And we could show the teachers in the Building Blocks curriculum that there's a lot of richness that you should be doing in geometry.

Then we said, okay, we're ready. We'll scale up then, things look good. First scale up project, 36 teachers we randomly assigned to a business-as-usual control group. Then another group here that was the comparison group got a competitor's curriculum, ostensibly research-based. So those teachers in the middle got a brand-new curriculum, brand-new materials, and exactly, exactly, because we had them, one met on the third floor, one met on the fifth floor of our building, the same amount of professional development.

The difference is the comparison group just learned how to do the curriculum, exactly the professional development that those writers of that curriculum called for. Our group did a little bit of that, but spent most of their time learning about kids' thinking and how the activities were supposed to enhance that thinking. So we outscored the control group business-as-usual by more than a standard deviation, but we also significantly outscored the comparison group by about half a standard deviation.

We think the big difference, again, is not a new curriculum or the Hawthorne effect or any of that kind of stuff. It's that they learned how to individualize and meet the needs of all the kids because they understood the learning trajectories. Okay. More on that later. We also are scaling up now. I'll skip through this pretty quickly. Here is 167 teachers, Boston, Buffalo, or Nashville, the control group gained a lot. But once again, we outscored them with an effect size of about .72.

So we found out if you do it right and you provide professional development and you teach people about learning trajectories, you can scale up even when you go into a whole district rather than that. Here's our longitudinal results, incidentally. This shows something too, kind of, oh, I got to change that color. You can't see the purple. I'm going to have to outline it here. In pre-K, which is the results I just showed you, these two groups are exactly the same, so they look the same.

Both these were Building Blocks groups. I didn't tell you that then we followed the kids, the control group. Here's, the orange is the group that was in the scale-up project called Triad, but they used the Building Blocks materials. But then we didn't do anything with the kindergarten and first grade teachers. And they start to lose because the kindergarten and first grade curricula don't build on, they don't assume any understanding, so they go back and start counting to five. Okay.

You can't see the purple, but it ends up, up here. They maintain more of the gains if this purple group works with us to also learn the learning trajectories. They didn't get a new curriculum. They didn't get anything except look at where the kids are already coming into you. Build on that. Don't just grind through the curricula you have. Those aren't demanding enough on the kids, basically, especially kids who have a good preparation. Okay?

So let's get by this. The result, the lesson here is use truly research-based education. And I don't care if it's a curriculum, a set of standards, an educational practice, a strategy. You got to be asking, what's your research basis for, and this, we did better than somebody else is not good enough. These kind of phases should be followed to truly trust it. Sir.

MAN: Yeah. I was wondering if you differentiated in that, in your studies between, you made a comment earlier about schools being low SES(?) and what-not. It seems to me, I mean it sounds like what you're saying is that schools that have money, are well-funded in math, we're like number two in the nation, and then, of course, the question comes up, then why mess with data? They must be doing something, and then whatever those schools are doing, we'll emulate that. There's something extraneous that's affecting . . . that's another question. But did you differentiate . . .

CLEMENTS: No. That's a really, really good question. Because of the nature of the funding from National Science Foundation and they used Department of Education IES, Institution of Educational Sciences, they're interested in helping kids who come from low resource communities. So all of this research was in urban settings for low resource communities. So, you know, the generalization is limited that way, and your point well taken. We didn't have any of those schools that are at the top. So maybe introducing our stuff would have made, for instance, very little difference if they're already performing and doing well.

MAN: So the . . .

CLEMENTS: So all that generalization is for these urban schools that don't have very good mathematics education in place, and that's the point of our grants. So good one. Keep going, keep going. What were . . .

MAN: I was thinking . . . what this, your thing with what those schools . . . a vast majority of the high-performing schools are, they might, they're not doing that. Or if they are, they might be doing it un-self-consciously. They don't know that they're doing similar things that you're recommending.

CLEMENTS: Yeah. And there are so many things that are happening in those schools. Because when you look at those schools, we're not just talking about the schools are doing better, right? You're talking about the parents have different kind of resources or you're talking about the communities do more with these kind of resources. So it's important not to abstract. What you just said makes sense.

Let's look at what those schools are doing. But in one way, we don't want to just look at what the schools are doing or what Singapore's doing and what Japan is doing so much as we want to say, what's the whole child experiencing in those countries, you know. So, for instance, one of the interesting facts is that the teachers that get bashed often the most in, you know, kind of right-wing pundit kind of things that said we should have charter schools because the public schools aren't doing well, they'll often point to the low performance of some of these urban schools and stuff like that and say they're not doing well.

But if you look at the research, in elementary school, there's a big difference between the high-performing and the low-performing kids, the high resource and the lower resource communities. Then during the school year, these teachers work their heads off and that difference starts to close. Then what happens? Summer vacation. It opens again. These kids actually stay the same or learn a little more mathematics over the summer. These kids sink.

The next year's teacher struggle, struggle, try to get that gap closed. Next summer, gap reopens. So it's, it's, what you said is true, but we also need a broader picture and a wider picture of the kind of things that are influencing those kids' education, because just emulating that and not changing the culture at large and what happens over the summer and everything else and the resources that the parents have probably won't be enough. Anybody else then? Either questions or comments on that? Agreements, disagreements?

Okay. Let's go to the next one. Informal and school mathematics. Really important. What goes wrong with kids very often, and especially from low resource communities or kids with special needs, is that they don't build or they lose the connection between their informal knowledge and reasoning and school mathematics. Okay. So let me give you an example. Key, remember the kid who invented that subtraction algorithm, isn't alone. A lot of people don't use school algorithms, and this is, I'm trying to make a point too about that important develop, discuss, and use notion, right?

Jean Lave, an anthropologist who studies cognition in everyday life. So in one of her studies, she followed supermarkets around in the, shoppers around in the supermarket to write down everything they did mathematically. She wasn't stalking them or anything. They were part of the study, and she just walked by, beside them and wrote down everything they did. So if you, Jane, were in the supermarket and you had like a quart of something and a gallon of something, no unit pricing, and you were trying to figure out which was really the better deal, should I go with the gallon or get the quart?

And she would just write down what you did to figure that out. And, Natalie, if you were in the line going out, you had three items, you had a $20 bill, you didn't want to pull out the credit card. And you got a kid with you, they're wanting a candy bar. You're trying to think, am I still going to be under $20? She wrote down how you solved that problem, what you did. And then she did, she kind of surprised the people, because she got them together in a room like this. They thought they were done. And she said, I have one more thing for you. Okay?

I've just got some tests that I want you to take. And they're all moaning, saying we hated this in school, I can't believe you're giving us a math test. She says, it's just for the research, just help us. And she gave everybody a math test, but she had one more trick up her sleeve. She didn't give the same math test to everybody. Jane's first problem said, you're in a store, there's a quart of something for this much and a gallon something for this much, which is a better deal?

And Natalie got a question, you're in a line, exactly what she saw them do in the supermarket, that's what was on their test. So how'd they do? How they'd do on that test? Well, not so good. Even in New York State where every time we introduce a new test, we lower the passing grade to not embarrass ourselves, this still would have been failing. They got about 59% correct on this paper-and-pencil test.

Now in the supermarket, neither place did they have calculators. For the test, they at least had paper and pencil, and they were sitting down, right? So how'd they do in the supermarket? Here's the killer. Not one person got one thing wrong in the supermarket. Not one shopper made one mistake. How could that be? What do you think happened? Pardon.

WOMAN: Math anxiety.

CLEMENTS: Math anxiety. As soon as you start a test, it just blocks your thinking. It's true. As soon as you worry too much about that, that's what fills your working, you know, your working memory and the like. Anything more?

MAN: They're not connecting, you know, conceptual apparatus of math with . . .

CLEMENTS: Yeah. They're losing the link between their informal knowledge of this and quantity, because when she looked at the papers, what did Jane do? Oooh. Yeah. I remember how to solve this. Let G equal gallons, let Q equal quarts, 4G equals Q. No, no, no, no, 4Q equals G. Oh, I never knew how to do this. Likewise, Natalie lined up all the figures, started adding up all the pennies first, carried, didn't carry two, she carried one, she made a mistake. They marked her wrong. She didn't do that in the supermarket, right. What did you do in the supermarket?

WOMAN: Estimated . . .

CLEMENTS: Estimated, rounded. You didn't care about pennies. And Jane just used, again, some kind of internal sense of, well, about four times this, where would it go, how does that compare to gallons? Okay. Another one. True. Another one. Weightwatchers. I used to be on the Weightwatchers program a long time ago, long time ago, like 20 years ago. And I remember substitutions, because I would go to Friendly's Ice Cream and say, I think I'll have this hot fudge sundae. I think that's like one milk and one fruit.

So but this guy in this study, this guy in this study was being honest about substitutions. Okay. He had eaten this teeny piece of chicken, carefully weighed it. It was one-fourth of his protein allowance. Now he looks in the book. He's going to eat cottage cheese for dinner, but he knew he had eaten one-fourth of his protein allowance. He looks up in the book. The books says two-thirds of a cup of cottage cheese is what you can have for protein. Oh, I got to figure out three-fourths of two-thirds of a cup of cottage cheese. Thinks for a minute because, I don't know about you, but you talk about test anxiety.

If somebody's watching me, I can't even add a column of figures. I just get, you know, so he's got an anthropologist who's like, well, how are you going to do that? You know, looking over his shoulder. But he smiles, gets out his measuring cup. Two-thirds of a cup of cottage cheese, pours it out on the counter, fashions it into a nice pie shape, cuts it, kathunk, kathunk, scrapes away one-fourth, puts that away. And what's he got? Three-fourths of two-thirds of a cup of cottage cheese.

So it says two things, these stories, I think. Number one, it says people don't use school-based algorithms all the time when they solve these problems. And maybe more importantly, it says that if they forget the school-based solution, if they understand the language of mathematics, it's kind of a recipe and not only states the problem, but for those who understand it, it suggests solution strategies if you've forgotten or just don't want to use the school-based algorithm. Okay.

So when I start with that, a lot of people say yeah, yeah, yeah. But basic facts, that's got to be straight memory, and we got to just have kids memorize it, right? Everybody's been saying that. This is a true Babylonian tablet. Okay. Six thousand years ago, this was a tablet that was found in a household. Okay. If I explain it a little bit, it's pretty easy to see what it is. One corresponds to nine. Two over here corresponds to oh, that must be one, ten, and eight. Oh, it's nines multiplication tablet. Okay. Three nines is two tens and seven. Four nines is three tens and six, right?

So picture it, guys. Babylon 6,000 years ago, some little kid is sitting down on the floor, and his mother says, get on the clay tablet, put down your ball. Okay. Facts have always been a concern. Early elementary people really worry the kids can't retain, and incidentally, kids with special needs, one of the first identifiers and strongest identifiers is they don't retain these kind of basic facts, right? But also, you guys who teach the upper grades, you want more than retention.

You want kids to know relationships. They can't make change. They can't generalize. Seven times 3 equals 21, so 7 times 4 equals, and they're lost. Students need a way into it. Relationships and strategies are important to everybody. How do we do it then? How do we do it? Respecting kids' thinking, but what do we do? Well, California took an approach. Okay.

This is what they did. They said, if you want to sell, you might remember California had a fairly NCTM reform-friendly set of standards, and then they got taken over by a very conservative side of the math wars, and they changed all their standards, and they said, listen, you want to sell some? It wasn't just policy, it was law. You want to sell a curriculum in California, you go to do at least two things. They had a whole list, but I'm going to just emphasize these two.

Number one, all kids have to memorize all their basic facts by the end of first grade. That's it. Secondly, you can't provide any support for kids to memorize or learn their basic facts in second grade because they should already know it, right? How'd that work out for them? Well, the title of the slide should tell you that, right? Twenty-six percent of the kids, a quarter of the kids knew seven, or 50% or more of the basic facts. A quarter of them knew a half. Only 7% met the 80% or above criterion for, 7%. This is what they were aiming for, and they succeeded with 7% of the kids.

Moreover, the more, this was a study, really interesting study by Brown and colleagues that looked at tens of thousands of these kids and their teachers and their teachers' practices, if you use the new state-approved textbook that followed those laws, you were less likely to have kids that met the criteria. If you used timed tests, you were less likely to have kids that could do timed tests.

It's really contradictory to everything lay people think about education. In the United States, we tend to think open the brain, pour in the facts. If you want them to memorize, go for it. Go for memorization, timed test and things like that. Didn't work, didn't work. Okay. Drill without meaning, drill without developing strategies and relationships actually hurts kids. It not only is ineffective, it hurts kids.

What did work? They looked at that too. Here's some things that worked, what we call conceptual subitizing. Oh, it was spelled wrong there. You know, having images, so eight plus five is up there, eight plus five, you know, you have an image of those numbers and of five and tens. And you can think, I'm going to take two of these and move them over and make it ten, you know. And then that strategy, counting strategies, counting on, you know, and stuff. And then, especially break apart to make ten.

The most effective first grade teaching was break apart to make tens. In other words, and let me, you know, this kind of illustrates that, eight and five, how do you do it? You break apart five in order to make that eight into ten, you see what you have left and that, right? Now, yeah, go, go.

WOMAN: Some . . . and they are taxing their working memories . . .

CLEMENTS: I agree with him completely for kids who have gone through and really understood the strategies and the meaning. This is the trouble is that we often get in these false dichotomies where you say, is it just going to be memorization, or is it going to be strategies, you know. It's got to be each, a learning trajectory says each in its own time. So what I'm thinking, what I'm saying here is that for the average first grader, it can be a fourth grader if that's the level they're on.

So I'm just saying the average normally developing first grader, their problem solving is at the level of thinking about these strategies and thinking. This research, I think, and a lot of other research, but this particular study is an illustration that if you try to not tax their working memory by just having them repeat 8 plus 5 equals 13 and remember it, it's gone. You saw the results. Seven percent of the kids knew it. They've been doing it all year. They weren't basing it on relationships at all.

But if you have a normally developing, again, end-of-year second grader, end-of-year third grader, who has to say, oh, 8 plus 5, 8, 9, 10, 11, then they can't do the multi-digit stuff. So each in its own time. What I'm talking about is developing this, how we develop, get up to the fluency. Then you need, actually I had that but kind of skipped it, you need intelligent practice then at that point.

After they develop the strategies, you absolutely need a lot of repetition, but it can be intelligent repetition. And I've got an activity for you to do in just a couple minutes to illustrate that. Does that answer you, or are you still thinking I'm dodging the question? I certainly don't want to do that. Is that, sir?

MAN: Don't some kids become more efficient though with their strategies? In other words, if you're teaching kids basic multiplication facts and you're teaching them how to count by twos, threes, fours and so on, they start after a while, three times two, they count by two three times. And then after a while, they begin to, that strategy becomes automatic and then the fact kind of becomes automatic over time.

CLEMENTS: I couldn't have said it better, and I got these cool slides that have these MRI images of the brain, and I'm not going to take the time to try to find those. But it's so cool. What he's saying is absolutely true. You don't stop doing strategies and start doing look-up in your brain. Let me just tell you that study and what they found out real quickly. I'll probably mess it all up. But very quickly, say you, you're used to saying 8 plus 5, I think either 8, 9, 10, 11, 12, 13, or I think 8, take off 2, I got 10, now 13.

When you get to memorize the basic facts, what do you do? You don't actually memorize eight plus five equals, and you've got a little cell here that memorizes that, you go pluck that out. That develops. In fact, when they do MRIs of fluent adults or older students, you know, that little part of the brain that ties an 8, a 5, and a 13 together is activated. But so are lots of other centers in the brain. For instance, the strategy, break apart and do that, is running over here, but fast and automatically.

They don't have to think about it. It's not taking up any working memory. It's a routine. Just like when you're driving a car. When I moved my houses, I ended up at my old house 10% of the time. Oh, crap, that's 30 minutes away from the new house. But it wasn't that I had no cognition to drive there. It was that my working memory was on other stuff, right? So that part is actually activated and does more than the three digits.

You know, the 8, 5, 13 connection. You know why? Because when eight and five, those two digits, even though you say eight plus five, when you hear eight plus five, you know what else lights up? Forty, right, and three, and I should have used seven plus eight because then also activated strongly is nine, right? So the string of digits is connected to a bunch of things. It's connected to multiplication, subtraction, the sequence of numbers of and everything else.

So you might say, wait, it's plus. They should only look up one. That's not how the mind works. The mind works that they see that eight and five, as in my example. I always use eight and seven, so I'm screwed up here. The 8 and 5 connects to 40, connects to 3 and connects to 13. The strategy, though, is tied right to that addition situation, so that's lit up. The front of the brain is lit up because it's trying to decide which strategy to use.

And in the very back of the brain, guess what's lit up? A number line that estimates not with perfect precision going out about 8, going out about 13, or 5 more, and what is lit up in your mind there? Thirteen, but also 12, a little bit 11, also 14, a little bit 15, but you're about there. So when 40 activates, the back of the head goes, reaches out actually to the 40 and say, not you, that's way too much. And all that happens in 2/1000 of a second.

You don't look up eight plus three. Just like this gentleman said, you get more and more fluent in estimating in your number line, in your number sense, and in strategies which go underground. What we want to make automatic is far better to make strategies automatic than it is to make the facts automatic. Because when I add 87 plus 8, I don't want to access 7 plus 8 is 15, put down the 5 carry it. I want to go up to 90, figure out how much I got left quickly and go up to 95. I can do that easily. The strategy is more important than the fact in a way, in that way. It's more flexible. It does more for you.

So now I probably totally blew my schedule, but that was a great point and a good question, so thanks for that. I'm going to go a little quicker then, trying to catch up. You got kids who have special needs. They often, one thing they don't do is pick up the strategy with the rest of the kids, right. So you've got some normally developing kids. They just learn counting on. They hear other kids do it. They start doing it. You kind of give them some guidance, and they do it. If they don't do it, what do you do? If they're not starting to go beyond because your question, I'm sorry, what was your name?

WOMAN: Andrea.

CLEMENTS: Andrea. Andrea's question, you know, the worst strategy or things that block it up is when we're counting by ones forever, right? You know, so eight plus five, fine. One, two, three, four, five, six, seven, eight. One, two, three, four, five. One, two, three, right, you know, then you don't even have a chance of making that connection between the 8 and the 5 and the 13, because by the time you count from 1 up to 13, the 8 and 5 are gone, right?

So you don't make that connection. So that's the worst. So counting on helps a little there. We got to make sure our basic skills are there. One of the things kids have to know is how to count from any number. They don't. You know, I don't know about you, but like I've known the alphabet my whole life, but I can't start from any letter and go forward and backward. I can start at A. I'm so proud. And I can start at L. But if you ask me what's two letters after R, L, M, N, O, P, Q, R, S, T, T. But I can't start from R, and I certainly can't go backwards.

It's a hard skill. You got to make sure they got that. Then counter-intuitively, don't ask kids, can you count on from, what's three plus two? Do it by counting on. They don't need two for three plus two. But you ask them 22 plus 1, oh, let's think about that. And then they start doing it. So actually a harder problem can get some of those kids going on counting on.

And then, finally, if none of that works, you've got a special needs kids, here's a technique that the Russians, a guy named Davida invented that's extremely effective, gets almost all kids counting on. Takes some patience, usually one-on-one instruction. But it's really effective. I'm going to whip through it and see if it makes sense to you. You tell a kid six and four, you ask them to put out six and put out four. Okay. Now we have to add them. So what does the kid do? Of course, he starts counting one, two, three four, five, six.

You let him solve it until eventually when the child gets to six, you start saying, oh, yeah, that's what this says, six, there's six over here. You just point that out, right? And then the kid keeps doing that, but you keep interrupting him sooner, saying when he gets one, two, three, four, what are you doing to say when I get to, when you get to here? Six. That's what this says. Do that a few times until that becomes kind of routine.

And then eventually say, see, there were six here, and this one with the big, exaggerated jump gets what number? Seven, right. And, again, you repeat, right, whoop, whoop, whoop, whoop. And you repeat that again and again, interrupting sooner until the kid starts, what's this one going to be? Six. That's right. And so then what number does this one get? Seven. Okay. How many in all? Seven, 8, 9, 10.

A repetition of that works with almost all kids. Gets them doing that because somewhere they lost one of those conceptual steps, and that really helps. Let me tell you one other hint for kids, all kids, but especially kids with special needs need this kind of structured guidance sometimes. This is the BAMT strategy. This is how expert teachers in the U.S. and Japan teach the break-apart-to-make-ten strategy. First of all, they spend a long time, and you got to read our book or look at a, one of these presentations where I lay out all the steps to it and all the prerequisites are at our website.

Most of the presentations and articles are at this location, at the upper left there, the ubtriad.org, our research site. Just click on articles, and you can get most of this stuff, virtually all of this stuff. So what do they do though? They make sure the kids can count on. They make sure the kids know that 10 plus 3 equals 13. Easier in Japan, right, than it is in the United States, because in Japan, you count [speaking Japanese], literally, nine, ten, ten-one, ten-two, ten-three. Way easier than our language.

But both countries, you got to make sure kids, that just makes perfect sense to kids. Then you give them problems like six plus four plus five. Oh, six and four, oh, you've done a lot of work with sums to ten, sums of ten, you know, the pairs of digits that add to ten. And then six plus four is ten. Oh, and then I just add on the 15, and that's when you get to here. Okay. And you might write down what's nine plus four. And the class works together, but the teacher illustrates it on the board, saying, okay, so what do we need to make ten out of nine?

Notice they always use nine so it's most obvious to kids, we need one. So I'm going to write, that's a part of four, and then the four is separated into two partners, so to speak, one and three. Good, because that's what left. And then they circle this and say, so that made ten. And what's ten and three? You guys all know that. A lot of repetition of that and then practice, practice, practice. And if the kids count on, don't use this in practice, so be it. The teacher just keeps bringing them back to it.

So eventually, the broad use of lots of different strategies, counting by one, counting on, doubles plus one, all that kind of stuff, more and more kids say, wow, this is a little faster than mine. I'm going to use this most of the time, right? And then they move to adding eight and adding seven to show it. And it really helps because I love this Sawyer, a mathematician back in the '50s, said the most depressing thing about arithmetic badly taught is that it destroys a child's intellect and, to some degree, his integrity.

Before they're taught arithmetic, children will not give their assent to utter nonsense, but afterwards they will. That's arithmetic badly taught. If we do it well, you get some interesting thinking.

[Videotape played]

CLEMENTS: Why didn't he know this one, do you think? How would you explain, it starts what?

WOMAN: . . . you know, it's . . .

CLEMENTS: Yeah, but he solved the subtraction problem. That's just an addition problem. What did . . .

MAN: . . . talks that way either.

CLEMENTS: There's that too, right?

MAN: Yeah.

CLEMENTS: Did you hear what he said? She said, you know, the sum word is tough, and he's saying you wouldn't really use that in natural language. So that's construction and presentation of the situation.

WOMAN: But he actually answers her question.

CLEMENTS: Oh, sum, yeah, exactly. That's what you just said. What more do you want from me? But he seemed to know he was reaching, reaching for things, right? You know, seven, one, one. I mean, he just started the typical strategy to throw out some numbers. Let's move on, you know. Sir.

MAN: I think . . . the problem was presented with gave away, which it sounds like take away, so . . .

CLEMENTS: Right.

MAN: . . . but the solution is . . . so as long as he had some help on . . .

CLEMENTS: He can't see the relationship. You might say that the sum is like, needs an algebraic understanding, like an X. You might just say he needs a part/part/whole notion that he can put these things. Where's the parts, where's the whole? He can't figure it out. The first problem, she presented one situation at a time, so he could build this, then go to the next one, then go to the next one. He can't here.

Teachers who know the learning trajectories for addition and subtraction would know exactly where this kid is and the type of problem that they could solve that was one little level above, but not the ones that are three levels above and that he needs some intermediate work before he builds up that algebraic or that part/part/whole kind of understanding. Okay? So there is a really good learning trajectory for addition and subtraction that we know. Teachers need to have access to that, not only on paper and a curriculum, but in their heads to be able to look at kids and work with them dynamically about where it is.

Okay. One other thing I wanted to say about learning trajectories is, wasn't it interesting, the old view of mathematics would be, you know, kindergarten, you count. Then first grade, you put away counting, and you do addition, subtraction. What was harder for him? The counting. How many times did he try to count to 13? I've seen like hundreds of times. Seven. Seven times he tried to count to 13. How many attempts did it take it to subtract? One.

Learning trajectory view of learning, we understand that these are separate trajectories, counting and addition, subtraction. Both start at one or two years of age, even addition, subtraction. Both grow independently and gradually become more and more integrated, making a strong, structural, conceptual whole. But both need to develop. Some kids might be a little father ahead on one, a little farther on the other, but you don't stop one, or you don't just do one. You need to meet both of those.

Okay. This next little girl. Boy, when I first saw the tape of her, these are beginning first graders, and I thought, oh, let her go back to her classroom. She doesn't seem to know how to use the blocks. She just kind of sits there, and then she surprised me a little bit. Let's see her.

[Videotape played]

CLEMENTS: Whoa. Whereupon my students say, play that again, what did she say, right? Interesting. You know, it wouldn't help her for her first grade teacher now to say you got to use the blocks, I don't think. Do you think it would help if her first grade teacher said, you don't, you've got to memorize 12 minus 5? I would say, no.

I would say she's got a powerful way of breaking numbers apart and putting them together on a base of ten and five, which research says is very important for kids to do, that help, will help her add 87 plus 8 better than if she memorizes 12 minus 5. And it's that strategy of breaking numbers apart that has to be automatic, not just a string of numbers memorized. Okay.

All right. So let's, and unfortunately, guys, unfortunately, you got to convince your colleagues too. Because kids can learn these things and then learn how not to think. Here's a little boy, Jamie, in a research study. He used inquiry-based kind of programming in which he was inventing his own strategies for addition and subtraction and other things throughout second grade.

He goes to third grade, and in eight weeks of conventional instruction, he lost his connection between school math and his informal math. He not only lost that connection, he lost access to the informal mathematics. Okay. So he was given this problem. Whoop. Typo on me. It's just 500 minus 251. No decimal point. Sorry about that. Written vertically. And he added, and then he realized his mistake. Only when asked to explain, oh, that's why I missed four on my test, I did plus instead of minuses, right.

So then the interviewer looked at his solution and started talking to him and said, Jamie said, yeah, that's right. Well, you know, there's a zero. You can't say one minus zero or zero minus five. So you have to take it, like numbers off the number that has more than that number, and then you took one off that. Off the five, yeah, all right. But I took two off right away. I put one in each. That would make it ten. Okay. You took two off the five, and that's why you have a three up here? Yeah.

And you put one on this zero and one on that zero. Yeah, and I made it to ten, and ten minus one is nine, ten minus five is five. Uh-huh. And three minus two is one. Okay. I see how you did that. Now can you think of another way to do that problem? No, not really. I can't. No. In second grade, he solved four-digit problems in his head. Now he can't think of a way. No? No.

So when you do subtraction, would you always do it that way? Probably, yeah. I don't know any other way I would do it. He knew it for a year. Eight weeks of conventional instruction, he doesn't know it anymore. What do we do for kids who have seemed to lost it, loose it? Hopefully, we can convince teachers not to let them lose it, keep doing mental math, keep doing alternate ways of solving it. Connect the informal strategies they have to the standard algorithm so the two aren't two different worlds.

But also pose non-routines, or regular problems in a non-routine way. If they don't respond to that, say, I bet I can do it faster than you. Race against calculators. One-half of the class got calculators, one-half of the class has to do it in their heads. Then give them problems like 1,293 plus 2, and the calculator people have to key in every digit, right? There's strategies you can use to keep kids thinking. Give them relational thinking problems, 102 plus 39 plus 98. Or 200 minus 198. One group has do it on, key in everything on the calculator. One group has to write the paper-and-pencil algorithm. One group gets to think about it. Who does it faster? Right. Those kind of things.

We need to, I'm going to skip this. We need to keep kids thinking. I wish I would have been this smart. I invented some things, simple things in elementary school, but I never thought of this one. You know, this has got to be one of the hardest subtraction problems, right, just like Jamie faced, right? Now look at these kids.

[Videotape played]

CLEMENTS: That's so smart, right? Why would you go through all that crossing out, regrouping stuff when a little thinking, much more dependable, much more efficient and more effective than the standard way of doing that kind of thing? Remember the international comparisons we were making before and talking about before? Those Japanese teachers at the top of the TIMSS? One of the things they have found is that Japanese teachers do ask more higher order questions than the average American teacher, right? But there's more to it than that.

They ask more questions about students' solution strategies. U.S. teachers, when they ask questions, ask questions about the solutions they demonstrated on the board. That's a big difference, right? We need to get into why procedures work. We need to talk about why the multiplication algorithm works and tie it to the properties of number and to visual notions because, once again, we're down here on the development of a rationale on the part of kids compared to other countries, right?

And that, of course, generates then ideas for how algebraic kind of stuff works and all. Okay. So we agree with the National Math Advisory Panel that although it was mostly actually packed with fairly conservative voices, fairly anti-reform voices because it was a politically-appointed panel, not a research-appointed panel. They still agreed. The curriculum was simultaneously developed, conceptual understanding, computational fluency, and problem-solving skills. Okay. There's no argument about that anymore.

So the lesson, it took me a long time to get to the end of it, is connect informal and school mathematics right through. Listen, you might think, well, I'm working with special needs kids, they can't invent these things. But I gave you a few strategies for this. And Art Baroody's research clearly shows they may be slow in learning it, they can invent their own given the right guidance. Okay. And I'll return to that in a minute.

But right now, I've been talking a long time, so I got another problem for you that kind of illustrates this whole notion of connecting informal and school mathematics of giving intelligent practice. I love Bob Wertz. Way back in the '50s, he came up with the idea. He said, there's different kinds of practice. He says, there's practice at the drill level. Some of that's good at the right time. There's practice at the applications level. That's like word problems. That's good too. But he said, my favorite is practice at the problem-solving level.

So he meant problems like this. You might know this problem. If you do, you'll just be an expert. If you don't, give it a try. Use four fours together with all the arithmetic signs and any amount of parentheses you want to try to build every number from zero to twenty. So, for instance, what's one easy one? Four plus four plus four plus four, right? We got 16. Well, how about what would four plus four minus four minus four be? Zero. We got zero. Okay. Work with the people at your table. See how many integers, whole numbers, from 0 to 20 you can come up with doing four 4s. Go ahead.

Anybody got a nice one that was difficult for them and they're kind of proud that they got it? Of course, then if it's easy for somebody else, you feel embarrassed, right? So we don't have to have any criteria. Who could just tell us some of the numbers you got? Natalie, help me out. What do you got there?

WOMAN: Four and four-tenths divided by four and four-tenths equals one.

CLEMENTS: Four and four-tenths, so 4.4…

WOMAN: . . . divided by

CLEMENTS: Divided by 4.4, a fancy way to get 1. Very nice. Okay. I didn't get you, but that's great. Okay. Esoteric solutions strategies. I love it. Who's got another number? Anybody got two then? Let's try to climb up. Anybody, you got two? You got two?

WOMAN: Yeah, I did.

CLEMENTS: How'd you do it?

WOMAN: Now I got to find it. Okay. I think I did. I did parentheses around four divided by four and get one. And then I did, the second parentheses you have four divided by four again. I got one, then I added the two.

CLEMENTS: Great. Did you do it the same way?

WOMAN: Yeah, but . . .

CLEMENTS: No.

WOMAN: Four fours plus four fours.

CLEMENTS: Four fours plus four fours, so you just took it as fractions. Same notion. Different way to represent it. Three. Anybody got three?

WOMAN: . . .

CLEMENTS: How'd you do three, ladies?

WOMAN: . . . numerator, four plus four plus four and all divided by four.

CLEMENTS: Cool. So they got 12, made it a fraction or just divided by 4, however you want to think about it. Four. Four's hard. Who got four? You'd think four would be easy.

WOMAN: . . .

CLEMENTS: You got it? How'd you do it?

WOMAN: . . . four . . .

CLEMENTS: Clever. Okay. You got it? So she multiplied four times the difference of two other fours. Zero wipes out that four. Just adds that last four gets four. Smart. Good. Good. Cool. Anybody else? Five? Go, go.

WOMAN: Okay. Four plus parentheses four plus four. That's . . . 20, divided by 4 is 1.

CLEMENTS: Cool. All right. Same, same, you got it the same way, huh? Anybody six? Boy, this is a good group. I never get a constant string like this. This is great. Go ahead.

WOMAN: I did parentheses four plus four, divide by four, plus four.

CLEMENTS: Cool. So four plus four, you get eight, divide by four, two, plus four. And we could keep going, right? And the point is what's developed with four fours? I hope you see there's skills, there's reasoning with numbers, numbers sense, a groundwork for later algebraic thinking. This is what I meant by practice at the problem-solving level. And you can change it all kinds of ways.

If you've got kids that aren't quite as good, use one, two, three, four, or any other combination. You're working on your sevens, multiplication or something, use sevens. Pick another digit. Use only the digit four, but you can use any number of them for kids who need a little help getting started and stuff like that, right.

Okay. Good. So connecting formal and informal school math, making strategies more the center. Oh, I didn't tell you one other study result for, especially those of you that are worried about kids should be fluent, and I'm worried about that too. We're just saying there's a different way towards fluency.

Bob Sieglar, Carnegie Mellon, has found that no matter what age kid, early childhood up through elementary, the number of different strategies they can employ in solving both single and multi-digit problems is the highest predictor of how they'll do in later mathematics. So it's not just, you know, they got to do strategies. It's that strategies have a particular power to make fluency more powerful and more likely and to give them lots of number sense and reasoning skills that then do other things for them.

So let's look at geometry, but I got a geometry test for you to start with. Okay. So just take any, any handout. Oh, there's no blank pages on this one. They were good about saving paper. You got a piece of blank paper? If you don't, find a little section on any piece of paper because it's very simple. All I need you to do to start this geometry test is in four separate places on the paper, draw a square, a circle, a triangle, and a squiggly line.

Draw a square somewhere, then a circle separate, a separate triangle, and a separate squiggly line. You all set? Okay. Now this next part of the test is very important that you do this right. That is, you have to do it quickly without thinking. Okay. Don't think about it, just react. Put an X on the one that feels most like you. Go, go, go, go. Okay. That's the whole test. And as you might guess, it's not really a geometry test, it's a personality test. So let's see how you did on this.

Okay. Let's see how the test did in predicting. Ready? Which you choose is supposed to tell you a lot about yourself. All right. We'll see how well it does. If you chose the square, if you chose the square, you're an organized person, you like lists, and you like checking things off your list, and you might be a little on the conservative side. If you chose the triangle, you're leaders, you want to know what the bottom line is. You might hate games like this, and you might be a little bit bossy.

If you chose the circle, you're nice people. And the only thing that psychologists tell you is, don't give yourself an ulcer trying to make everybody happy. Okay. And if you chose the squiggly line, you know, the old saying that the lights are on, but nobody's, no, just kidding, just kidding, just kidding. That's not true. Squiggly lines are creative people. Their desks might have piles on them of stuff all over. Their car's filled with math ed materials, right? Might like dirty jokes.

Okay. That's not the end. We're all in education, so we all have to do meetings, right. But different people react to meetings differently. If you chose the square, it's likely that you check it off your list, but you might not go. If you're a triangle, you sit in the back and say, I'll do what I want at this meeting, but you might not go. If you're a circle, you go for the food and the company. And if you're a squiggly line, you didn't know there was a meeting. So you got to get one of our square friends to tell you about these meetings.

All right. I hope I don't offend anybody, getting a little personal here. Love life. All right. Here we go. If you chose the square, it's same time and same place. All right. Triangles are the best lovers, and the only problem is they tell you that. Circle is whatever pleases you. All right. And the squiggly line is black leather and lace in the closet. All right. So, obviously, this test has no validity or reliability. The real message of it, at school, at home, in our community, we need all these shapes, and we have to deal with them, kids and adults.

And shapes are a large part of our metaphorical lives. I like to say without geometry, life is pointless, but that's an even worse joke. I know, I know. It's a groaner. Okay. What does the research actually say? We don't do very well in geometry and measurement. We score in those international comparisons about the lowest on that. One shocking study found that first graders followed longitudinally. When they were in first grade, they were more likely to differentiate one polygon from another by counting its size and angles than they were when they got to third grade. We teach it out of them with stuff like this, right?

Now, hey, at least the top triangle doesn't have a horizontal base. Well, that's good. But, you know how they put a, the publishers put a dotted line around one thing so you can see how you're, what you're supposed to do. You're supposed to put a ring around the triangles and stuff like that. Look what they chose for that. It's got a hook, right? Hey, my mathematician friends will all say, numbers are important, arithmetic's important, geometry's important. But at its core, mathematics is about precision of thinking and reasoning. This is just sloppy, right?

I mean, musical triangle, actually they have one pictured close. Most of them, what, aren't closed, and there's no angles in musical triangles and . . .

MAN: Just to add to what you're saying . . .

CLEMENTS: Yeah, go.

MAN: I watched a first grade teacher teaching kids the difference between cone and a pyramid, and she had geometric shapes and she says, a cone will roll. Then she gave them the worksheet, and I saw the little girl trying to . . . the cone on the paper.

CLEMENTS: Yeah, it's too much with, we go to paper too fast with 3D, that's for sure. My son got a star on this. He did perfectly, but he circled here what? A pentagon. He might have been looking at the two triangular sails, but he circled the pentagon. Nobody talked to him about that. 3D and 2D here completely confuse, then what's this sandwich doing here? You're talking precision and reasoning and thinking. We don't need that stuff. We got to get rid of it kind of stuff, that lousy stuff.

We got to do basic shape-naming better, but we also have to move beyond basic shape-naming to where kids are talking, pre-K through eight and above, about shapes' attributes, analyzing them, talking about them, describing them, building mental image, transforming them, and composing and decomposing them, right?

So we have kids, for instance, build shapes from parts both physically with sticks and string and clay and everything else and other manipulatives and on the computer where they can learn about the transformations more explicitly. We give them a challenge, try go make a trapezoid, and then they can't draw it or pick it. They have to give the properties that they think it has, and woe be it to the kid who thinks they're going to make a rectangle by clicking two long sides and two short sides, because they end up with non-rectangular parallelograms.

They end up with chevrons, you know, deltoids, like kite shapes and everything else, right? Because, of course, we've programmed it to give you the least likely case in many cases. And you keep on clicking on change shape, change shape, so you can keep on seeing a lot of shapes that follow your properties, but you're trying to narrow it down to the shape you want.

So I don't have much time for this today, but the main lesson is, let's make sure we include geometry and spatial thinking. It's critical for all ages. It provides meaningful and motivating setting for number, for logic, for later calculus. There's little to lose and a lot to gain from doing more geometry. Finally, I got to kick it up a notch because I'm almost done here.

Picture kids doing students, we often like manipulatives. One of the things we often don't do enough of is to realize that computer manipulatives can sometimes be just as effective or more effective because the math isn't in the object. It's in the thinking about the object. And as long as kids are making sense of it, it can make more sense. So physical-based ten blocks are fine, but on the computer, for instance, you can see not only the blocks, but you can see, and I know it's hard to see in the back, this just gives the kind of expanded notation, three 1,000's, five 10's and two 1's and then over there 3052, the conical kind of representation.

And if you add a block, then all these change. Linked representations are what we need in kids' heads because that's what's gives meaning to these things. So even when they subtract, for instance, if they're subtracting and if they regroup, if they use physical blocks, they have to take 100 and put it aside and get ten 1's. That's close, not exactly what we want the kids to do. We want them to think of breaking that 100 into ten 10's, that that same 100 separates, and you can't do that with the physical blocks. You can with these kind of representations.

Do kids need this? They sure do. Jim Hebert walked around a classroom where they were sure the teacher said they know their base ten. He just asked kids to use base 10 blocks and make him 35, and a whole bunch of the kids in the classroom showed him three 1's and five 10's, and pointed three 1's and said 30, pointed to four 10's. They hadn't constructed the relationships. Now computers aren't magic, but if a kid did that on a computer, at least the computer would say 53, and they'd have to think, what am I doing here? What's going on here, you know?

I don't have time for this, but I wanted to show you, geometric sketchpad and other kind of geometry kind of things can present properties. So in this computer program, what you do is you specify that you want a parallelogram, and it maintains its properties. You can turn it, you can change its size, you can change its angles. You can't make it into a trapezoid that has one and only one pair of sides. It's restricted.

So kids get a physical, virtually physical kind of experience with the hierarchical classification of geometric shapes and the propagation of properties in these kind of environments. That's why, as the National Math Panel, this is the report I actually wrote that concluded that engaging kids with high quality, very important caveat, implemented well, another important caveat, can raise students' achievement compared to other kind of things.

Okay. Let's move on to my final kind of thing. Low achievers, students with special needs. We have a good report on that in the National Math Panel, and you can dig beneath the executive summary and see these long, long chapters that give you a lot more details. But just very briefly, I want to emphasize that explicit methods in instruction, when we went into it, a lot of people said, yeah, I know what you're going to find with kids with special needs, you know, or low, really low-performing kids, direct instruction. It didn't turn out that way.

Direct instruction's fine, but direct instruction, which often means just telling the kids and stuff like that, was not as powerful as what we called then explicit instruction. It includes direct instruction, but it also includes problem-solving models, clearly orchestrated example, concrete objects to illustrate the kind of abstract relationship and lots of think-alouds, not just by the teacher, but by the kid as they solve problems and the teacher listens explicitly to these solution strategies and tries to work with that and develop that.

That's really important because often kids are classified as having a learning disability when they've only experienced not so good conventional instruction. Some experts estimate that as many as 80% of the kids in the early grades are misclassified when they haven't had the opportunity to do this. One of the things we need to do for those kids and for all kids is do more formative assessment. It especially helps those kids who are low performing because it develops the higher order meta-cognitive skills they need, right?

So let's talk about formative assessment because Julie Sarama, wife and colleague of mine, who's working on all these projects, and I think that the research is pretty clear, that the missing piece in most formative assessment is knowledge on the part of the curriculum writers and the teacher of these learning trajectories. You need a path to know where the learning's going and how kids, the bumps that kids, the obstacles that kids run into, and the likely activities that are going to facilitate the next level of thinking.

So let me give you one more example quickly of an early childhood, because that's where Julie and I have been doing our work lately, a trajectory for composing geometric shapes. I already gave you an arithmetic example of those two little kids. It was only a little part of an example, but that's one example. Here's another one where they're composing and decomposing geometric shapes. And this little girl, can you see her solution here, doesn't seem to have a whole lot of confidence there in composing shapes.

She can barely get the shapes near the outline, and she's not even matching shapes to the outline. Compare that to this little girl. And I'm going to stop the video so I can explain real quickly, she is right now, right over here, use it, working on this puzzle. She's trying to get that orange square from the pattern box set in where you and I can see the blue rhombus is really needed. Watch how she goes about it. Okay. She first tries to push it into shape, place, and then she knocks the other shapes out of position.

Then she fixes those, but that knocks the square out of position. She tries to hold those and push the square into there, and that doesn't work. She finally gets up and, lucky for her, right next to her hand is a blue rhombus, and it works out. And if you only saw her final product, you would say she's fine, but you got to be aware of the process. This is what we want to develop in kids.

Okay. Watch this little girl, still only a five-year-old kid, but watch how she's using, doing the same robot puzzle. Watch how intentionally she uses symmetry. She fills in one side. She gets the exact same piece, and she fills in the correlated place on the other side symmetrically, right, mirror symmetry. And, more important than that, watch the other kids at earlier levels of thinking would grab a shape and then put it on the puzzle and then turn it and move it and slide it into place. Where does she move the shapes? In the air.

By the time they touch the paper, they're in the exact right position already. Because she's reached the level in this learning trajectory where she can build, maintain, and manipulate shapes mentally. That's what we want, because then she has a powerful tool for geometry, for understanding area models of multiplication, for understanding area models of fractions, and for understanding architecture and the visual arts.

Okay. This is the kind of stuff we need to do. We need to get kids, so, so quick view, what do you instructionally? The first little girl might get something like this. They only touch at the vertices. It really helps her just match the shapes, right? But it gives her a clue. The second girl, clearly defined areas, so it's pretty easy, but they're touching at the sides. So she starts to build that up. And the third girl might get this or this one or there's a lot more tension has to be given to filling more ambiguous areas and looking at the angles. And eventually that little girl should get this.

Now it looks a little easier, but you got to know that after she finishes this one, the computer says, good job, takes off the shapes, and said, now you can't use those shapes again. Solve it a different way. So she has to substitute some shapes for others, and we always let the kid invent their own puzzles afterwards and solve those. For upper grades into middle school, they do different things. Here they do what we call super shape because here they only get one shape. They have to decompose that shape and then recompose it to solve the puzzle.

Here, for instance, look, they can even split that little pattern block triangle into two right triangles to solve that. And try one of these on your own. Our graduate assistants have trouble at this level because you get a, you get three squares. You cut one of them wrong, cut vertex to vertex or vertex to midpoint of a side, you cut one of them wrong and screw it up, you're done for, because you need to cut every one exactly right. It takes a lot of thinking, a lot of that.

Is this just for little kids? Is this just for elementary kids? Well, yes and no. It forms a good foundation, but kids need to be able to compose and decompose. Mike Battista and I have done all this work with what we call spatial structuring. Kids don't understand formula, area by giving them a formula. They misunderstand that all the time. They screw up perimeter and area. They have to understand area is composed of shapes composed into a unit called a row, and the unit iterated across the number of columns or vice versa. They have to understand to solve other kind of problems how to compose and decompose areas in different ways to use a formula.

To understand triangles, they might decompose that triangle into two pieces, put those together in a different way, and make sense of one-half base times height or duplicate the triangle. Put those two together. Good. Now it makes sense, one-half base times height, right? And even when we get into calculus, what is this but the composition of area. It's very important. So the lesson here is format of assessment is very important.

It enhances mathematics achievement we found in the National Math Panel, especially when information is used to determine the focus of instruction. Expert teachers offer advice to teachers, and computer-assisted instruction or peer tutoring is an effective component, too. One of the projects Julie and I are working on now is called The Team, and in The Team, we follow a three-part thing.

We assess the kids to know where they are in the learning trajectory, we get reports for kids on each learning trajectory, and then we apply that information by giving teachers a report that correlated to a variety of instructional materials so they can kind find lessons that help that kid right at the level of thinking that they're on. Okay. All done.

Let's just review real quick. What did we talk about? Lessons from research included the following. Gaps are striking, especially between kids from lower and higher resource community but also internationally. Less is more, not less mathematics but more time on key concepts.

Look at the curriculum focal points, compare those to the Common Core for your grade, and wait but keep looking for the learning progressions, the mathematical progressions that underlie the Common Core that are learning trajectories or at least pieces of them. A little warning, like I say, some of the progressions are written by people like me, and we've tried, I've tried my best to integrate the research and the mathematics. Some of them are written by mathematicians, and it doesn't say very much about actual learning.

It's more this topic, then this topic, then this topic. They're still helpful, but, you know, they're not everything I would like them to be. But they're going to be there, and they're going to be of some help for reflection.

Use truly research-based education, and I tried to give you a little snapshot of some ideas of criteria that you should use before you go out and spend any time or money because you got limited time and money, and we want to spend it on stuff that really makes a difference. Connect informal and school mathematics, build up those strategies on the way to fluency so kids have this absolute thing. And kids who have special needs are especially needing that kind of approach so that they have multiple ways to access and remember the basic facts and then multi-digit kind of solution strategies.

And, finally, include geometry and measurement in a more substantive way, meet the needs of all students, both by using manipulatives correctly, by using computer manipulatives and computer explicit instruction and the like, and by using formative assessment that relies on learning trajectories, which should be a basis for choosing curriculum for teaching and also for professional development. Phew. I'm okay. I got one more minute. Well, I got three, but I'll only use one.

One last research lesson that I'm so pleased that this study finally came out. I have believed this for years, and I have empirical evidence now that eating chocolate helps improve your math scores, all right? It's really true. It's got to be dark chocolate, so I don't know about the Mr. Goodbars here, but get the dark chocolate, the Hershey's dark chocolate, 30 minutes before a math test actually improves kids' math scores. It's true. Dark chocolate every day. There's our websites. Remember they're on the handout. Thank you for coming and spending so much time with me today. I'll see some of you at 12:30.